

Preventive Strike vs. False Targets in Defense Strategy

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Abstract: A defender allocates its resource between defending an object passively by deploying false targets, and striking preventively against an attacker seeking to destroy the object. With no preventive strike the defender allocates its entire resource to deploying false targets, which the attacker cannot distinguish from the genuine object. If the defender strikes preventively, the attacker's vulnerability depends on its protection and on the defender's resource allocated to the strike. If the attacker survives, the object's vulnerability depends on the attacker's revenge attack resource allocated to the attacked object. An optimization model is presented for making a decision about the efficiency of the preventive strike based on the estimated attack probability, dependent on a variety of model parameters. The optimal number of false targets to deploy and the optimal subset of targets to attack are determined.

Keywords: *Vulnerability, active defense, passive defense, attack, protection, contest intensity*

1. Introduction

Defending against external impacts and especially against intentional external impacts has attracted considerable research effort in recent years. One can distinguish between active and passive defense. Some measures aimed at mitigating the effect of external attacks, such as protective shields, are by their nature defensive. Other measures can generate active defense which means exerting effort when certain conditions are met. See [1] for a review of earlier research in this area, classifying according to system structure, defense measures, and attack tactics and circumstances. Two subcategories within defense measures are false targets and preventive strike. The preventive strike can be an effective measure of active defense aimed at destroying the potential attacker and, thus, preventing the defended object from destruction. However, the preventive strike can inflict a revenge strike which causes expenditure of the defender's resources that could be used for passive defense. Thus, the optimal balance between the passive and active defense can considerably improve the survivability of the defended object.

Earlier research has considered how a defender balances between protecting an object passively and striking preventively against an attacker, equipped with one or multiple attack facilities, seeking to destroy the object. In this paper the defender determines a balance between striking preventively and deploying false targets to distract the attacker. Unlike the previous works, we assume that the object already has some given protection and the defender distributes the remaining resource between the preventive strike and the false targets. The attacker cannot distinguish the genuine object from the false targets. First we assume that the attacker attacks all targets. Thereafter we assume that the attacker may attack a subset of the targets. In any attack against a group of targets the attacker distributes its effort evenly among the targets. This corresponds to the case when the attacker cannot direct the attack exactly against certain targets, but against a group of targets (for example, area coverage weapon attack against a group of separated targets).

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This paper analyzes the defender's objective of minimizing the probability of destruction of an object it controls. The object may be an asset, a collection of assets, an infrastructure, a country, *etc.* The defender's two strategies are to defend passively or strike preventively, and, if the latter, which resource fraction to allocate to the preventive strike.

It is sometimes suggested that attack is the best defense, but not always. This paper seeks to determine when it is optimal to stay on the defensive and await the blow, and when it is optimal to go on the offensive and strike preventively. Our focus is on the quite challenging defense optimization where the defender has a fixed resource which can be used passively or actively. The attacker has two resources, one resource that is used for attack, and one that is used to protect against a preventive strike by the defender.

As an example, consider an airborne bomber that has a mission to destroy some camouflaged object. To dissipate the bomb strike the defender deploys false targets. The bomber can detect the targets with a given probability. If the targets are detected the bomber distributes its load among a subset of targets it chooses. The defender can strike preventively using short range anti-aircraft missiles. As the missile launchers are located near the defended targets, the preventive strike reveals the locations of targets and, therefore, if the bomber, protected by an anti-missile system survives the strike, it attacks for certain. The defender has to make a choice between the passive defense (hoping that either it is not detected or that the object protection can survive the strike weakened by dissipation among several targets) and active defense (hoping that the attacker is destroyed). When the defender builds its defense it should decide how its limited budget is distributed between deploying the false targets and deploying anti-aircraft systems.

In this study we assume that the level of the object protection is a fixed exogenously given parameter, which means that the protection resource is independent from the resource distributed between the false targets and the preventive strike. We analyze how the object protection affects the decision regarding the PS in the presence of the FT deployment option.

Reference [5] states that a preventive war is "initiated in the belief that military conflict, while not imminent, is inevitable, and that to delay would involve greater risk." In contrast, preemption is defined as "an attack initiated on the basis of incontrovertible evidence that an enemy attack is imminent." According to these two definitions, the focus in this paper is on preventive strike, and not on preemption. In related research [2] draws on sociological and psychological research to consider risk governance, [3] introduces human factors into safety analysis, and [4] balances hard and soft issues for the performability of work teams.

Section 2 presents the model when the attacker attacks all targets. Section 3 assumes that the attacker attacks a subset of the targets. Section 4 illustrates the solution. Section 5 considers the conservative defense strategy under uncertain contest intensities. Section 6 concludes.

2. The Model (When the Attacker attacks all Targets)

Nomenclature

PS	preventive strike
FT	false target
r	total defender's resource
R	total attacker's offensive resource
t	effort with which the defended object is protected

H	number of FTs deployed by the defender
H^*	optimal value of H
Q	number of targets attacked at random by the attacker
Q_H^*	optimal value of Q given H FTs are deployed
D	attacker's protection resource
c	the cost of single false target
T	attacker's effort (resource) per attacked target
σ	ratio c/r between cost of FT and the total defender's resource
τ	ratio t/R between defender's protection effort and attacker's attack resource
δ	ratio D/r between attacker's protection effort and total defender's resource
h	maximal possible number of deployed FTs ($h=\lfloor 1/\sigma \rfloor$)
$v(H)$	conditional probability of the defended object destruction given it is attacked as well as H FTs
$V(H)$	conditional probability of the attacker's destruction given it is preventively attacked
m	contest intensity in strike against the attacker
μ	contest intensity in attack against the defender
z	estimated probability of attack against the defender if there is no preventive strike
z_{\min}	threshold value of attack probability (minimum estimated attack probability when the preventive strike is justified)
\tilde{P}	probability of destruction of the defended object in the case of no preventive strike
$P(H)$	probability of destruction of the defended object in the case of preventive strike against all targets
G	probability of destruction of the defended object in the revenge attack against Q randomly chosen targets
W	probability of destruction of the defended object under the optimal defense strategy

2.1. The Model

Suppose the defender anticipates an attack from the attacker. The attack can be directed against an object owned or controlled by the defender, or against the defender itself. The estimated probability of the attack is z . This probability can be elicited from intelligence data, expert judgments, and previous statistics (this paper considers z as an exogenously given fixed parameter). The defender can defend its object in two ways: implementing the preventive strike against the potential attacker (active defense) and deploying false targets to dissipate the attacker's resources (passive defense). In the case of the preventive strike, the defender distributes its resource r between strike effort and false targets. The cost of a single FT is c . Therefore, the maximal number of FTs that can be deployed given the defender's resource r is $\lfloor r/c \rfloor = \lfloor 1/\sigma \rfloor$, defined as the largest integer not greater than $1/\sigma$. If $H \leq \lfloor 1/\sigma \rfloor$ false targets are deployed, the remaining resource $r - Hc$ can be allocated to the preventive strike. If the attacker survives the preventive strike, it attacks the defender with probability 1 (revenge attack), evenly distributing its attack resource among all the targets. If $x=0$ (no preventive strike) the defender allocates its entire resource r into deploying $h = \lfloor 1/\sigma \rfloor$ FTs.

The defender has one free choice variable H , which implies the amount $r - Hc$ allocated to the preventive strike. Hence the choice about the preventive strike determines

the number of FTs deployed and vice versa. To focus explicitly on these strategic choices, we abstract away from the defender's protection which is exogenously given. First, the defender makes only one decision at one point in time, which either gives no preventive strike or a preventive strike with a resource allocation between the strike and FTs. This decision takes into account the magnitude of the attacker's attack which gets distributed across the genuine object and the FTs. Second, an alternative model where the defender can decide different protections before and after its preventive strike gets more complicated, and is unrealistic when the attacker's revenge attack comes so quickly that the defender does not have the time to change its protection. Especially protections such as bunkers, protective casings, and shields of various kinds in remote areas are not easily changed quickly. Consequently, first, we consider the cases when the deployment of ready FTs can be accomplished much more easily and quickly than protection enhancement. Second, we consider the cases when the protection is limited by technology (for example, moving targets such as tanks or airplanes cannot have unlimited protection and the decision about their protection is made by the designer, not by the field commander) and the only way to improve the defense of already protected object is to deploy FTs which can be done in battle conditions.

The attacker has no free choice variables. We thus analyze the defense optimization. The vulnerability (conditional probability of destruction in the case of attack) of the attacked object is determined by the common ratio form of the attacker-defender contest success function [6,7]

$$v = \frac{T^\mu}{T^\mu + t^\mu}, \quad (1)$$

where T is the attacker's effort, t is the defender's effort, $\partial v / \partial T \geq 0$, $\partial v / \partial t \leq 0$, and $\mu \geq 0$ is a parameter for the contest intensity. When $\mu=0$, t and T have no impact on v regardless of their size which gives vulnerability $v=0.5$ for any $T>0$ and $t>0$. When $0<\mu<1$, exerting more effort than one's opponent gives less advantage in terms of vulnerability than the proportionality of the agents' efforts specify. For example, $T=2$, $t=1$, $\mu=0.5$ gives $v=0.59 < 2/3$. When $\mu=1$, the efforts have proportional impact on the v . When $\mu>1$, exerting more effort than one's opponent gives more advantage in terms of vulnerability than the proportionality of the agents' efforts specify. For example, $T=2$, $t=1$, $\mu=2$ gives $v=0.8 > 2/3$. Finally, $\mu=\infty$ gives a step function where "winner-takes-all". The parameter μ can be illustrated by the history of warfare. Low intensity occurs in situations where neither the defender nor the attacker can get a significant upper hand. Examples are the time prior to cannons and modern fortifications in the fifteenth century, and entrenchment used with the machine gun in World War I [8]. High μ occurs when one or the other opponent more easily can get the upper hand. Airplanes, tanks, and mechanized infantry in World War II allowed both the offense and defense to concentrate firepower more rapidly, which intensified the effect of force superiority.

In the case of no preventive strike the defender allocates $h=\lfloor 1/\sigma \rfloor$ FTs. The attacker distributes its resource evenly among $h+1$ targets achieving the per-target effort

$T = \frac{R}{h+1}$. The probability of the destruction of the defended object \tilde{P} is

$$\tilde{P} = z v(h) = z \frac{T^\mu}{T^\mu + t^\mu} = \frac{z}{1 + ((h+1)t/R)^\mu} = \frac{z}{1 + (\tau(h+1))^\mu}. \quad (2)$$

In the case of preventive strike the defender exerts effort $r-Hc$ remaining after deploying H FTs into strike and the attacker exerts the effort D to defend its facility. The vulnerability of the attacker's facility is

$$V(H) = \frac{(r-Hc)^m}{(r-Hc)^m + D^m} = \frac{1}{1 + (\delta/(1-H\sigma))^m}, \quad 0 \leq H \leq h. \quad (3)$$

where m has the same interpretation as μ . In the revenge attack the attacker exerts the per-target effort $R/(H+1)$. The vulnerability of the defended object in the revenge attack given the attacker survives the preventive strike is

$$v(H) = \frac{1}{1 + ((H+1)t/R)^\mu}, \quad 0 \leq H \leq h. \quad (4)$$

In the case of preventive strike the probability of destruction of the defended object is

$$P(H) = (1-V(H))v(H) = \frac{1}{1 + ((r-H\sigma)/D)^m} \cdot \frac{1}{1 + ((H+1)t/R)^\mu} = \frac{1}{1 + ((1-H\sigma)/\delta)^m} \cdot \frac{1}{1 + ((H+1)\tau)^\mu}. \quad (5)$$

In our first formulation the defender has to choose H , while D , t , z and R are exogenously given.

2.2 Solving the Model

The defender optimizes its resource distribution in order to minimize the probability of destruction of the defended object: $H^* = \arg \min_{0 \leq H \leq r/c} (P(H))$. Comparing (5) and (2), the

preventive strike is justified if

$$P(H^*) < \tilde{P}, \text{ i.e., } \frac{1}{1 + ((1-H^*\sigma)/\delta)^m} \cdot \frac{1}{1 + ((H^*+1)\tau)^\mu} < \frac{z}{1 + ((h+1)\tau)^\mu}. \quad (6)$$

It follows from (6) that the defender should strike preventively if according to its estimates the probability of the attack against the defended object z exceeds the threshold value z_{\min} , where

$$z_{\min} = \frac{1}{1 + ((1-H^*\sigma)/\delta)^m} \cdot \frac{1 + ((h+1)\tau)^\mu}{1 + ((H^*+1)\tau)^\mu}. \quad (7)$$

The probability of the object destruction given the optimal defense strategy is $W = \min\{\tilde{P}, P(H^*)\}$.

We assume that the defender's evaluation of the probability z does not change between its choice of H and the attack. Thus, the decision regarding the preventive strike can be made simultaneously with choosing the value of H based on (6).

We now proceed to analyze the impact of the variation in the six parameters σ , δ , τ , z , m , μ on the decision variables H and Q , and on the dependent variables P , W , and z_{\min} .

2.3 Illustrating the Optimal Defender's Decisions

Fig. 1 presents the values of \tilde{P} (no PS) and P for $H=1$ and $H=2$ as functions of σ , when $\delta=0.5$, $\tau=1$, $z=0.8$, $m=\mu=1$. As the FT cost increases relative to the defender's resource, the probability of destruction increases. With no PS, the probability increases as a stepwise function. Indeed, in intervals when $h=\lfloor 1/\sigma \rfloor$ remains unchanged, σ does not affect \tilde{P} . Depending on σ as well as on the number of FTs H , the probability $P(H)$ can be lower or greater than \tilde{P} . The combination of stepwise increasing $\tilde{P}(\sigma)$ and

monotonically increasing $P(\sigma)$ causes situations when the difference $P(\sigma) - \tilde{P}(\sigma)$ changes its sign several times with increase of σ . For example, for $H=2$, $P(\sigma) > \tilde{P}(\sigma)$ when $\sigma \leq 0.16$, $P(\sigma) < \tilde{P}(\sigma)$ when $0.19 < \sigma \leq 0.44$, and again $P(\sigma) > \tilde{P}(\sigma)$ when $\sigma > 0.44$.

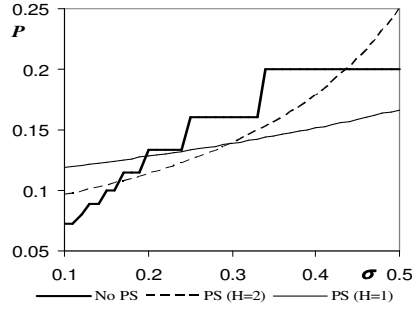


Figure 1: \tilde{P} and P for $H=1$ and $H=2$ as functions of σ , when $\delta=0.5$, $\tau=1$, $z=0.8$, $m=\mu=1$.

Figure 2 presents \tilde{P} for $z=0.8$ and $z=0.6$, $P(H^*)$ and corresponding values of h and H^* as functions of σ , when $\delta=0.5$, $\tau=1$, $m=\mu=1$. Depending on the relation between \tilde{P} and $P(H^*)$ the defender decides whether to strike preventively or not. In this example the defender avoids the PS when $z=0.6$ (except when $0.33 < \sigma < 0.39$, where \tilde{P} becomes slightly greater than $P(H^*)$) and always strikes preventively when $z=0.8$, which corresponds to high probability of the unprovoked attacker's strike. The defender always chooses $H^* < h$ in order to allocate some resources to the PS. Both H^* and h decrease in σ .

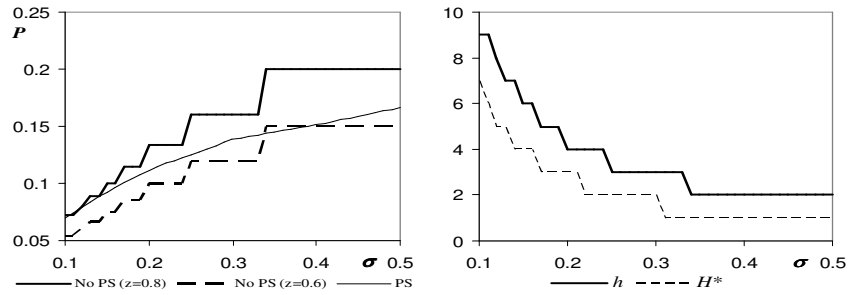


Figure 2: \tilde{P} for $z=0.8$ and $z=0.6$, $P(H^*)$ and corresponding values of h and H^* as functions of σ , when $\delta=0.5$, $\tau=1$, $m=\mu=1$.

Figure 3 presents \tilde{P} for $z=0.7$, $P(H^*)$, W and z_{\min} as functions of σ , when $\delta=0.5$, $\tau=1$, $m=\mu=1$.

It can be seen that z_{\min} (thick line) varies non-monotonically and the defender's choice of no PS vs. PS can change many times as σ increases. Indeed, when $z_{\min} > 0.7$ (where 0.7 is shown with a thin horizontal line) the defender avoids PS, and when $z_{\min} < 0.7$ the defender strikes preventively.

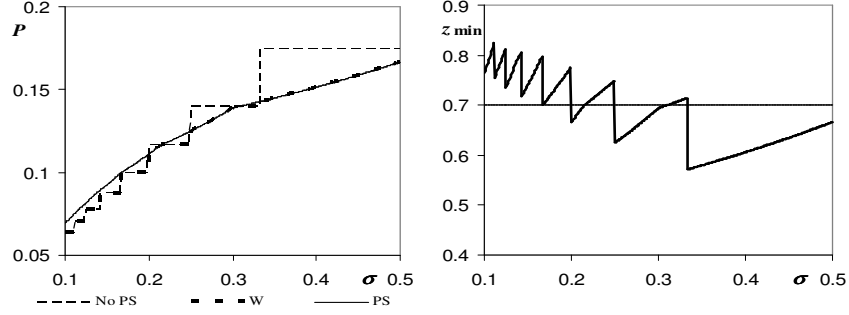


Figure 3: \tilde{P} for $z=0.7$, $P(H^*)$, W and z_{\min} as functions of σ , when $\delta=0.5$, $\tau=1$, $m=\mu=1$.

Figure 4 presents W for $z=0.6$, z_{\min} and H^* as functions of σ , for $\tau=1$, $m=\mu=1$ and different δ . The destruction probability W increases in σ , especially when the attacker enjoys a large resource advantage δ compared with the defender. z_{\min} has an overall decreasing trend except when δ is large. The non-monotonic behavior of z_{\min} makes the intuitive defender's decision about the optimal defense strategy difficult. H^* decreases in σ starting out at the largest level $H^*=9$ when the attacker enjoys a large $\delta=1.6$. The defender compensates for its resource inferiority by deploying many FTs.

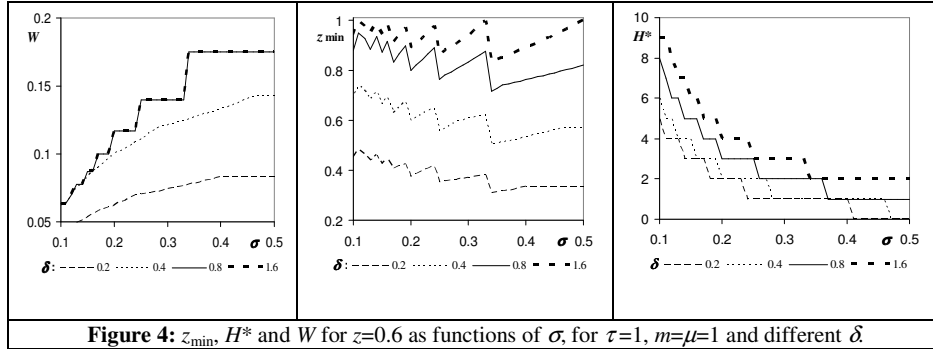


Figure 4: z_{\min} , H^* and W for $z=0.6$ as functions of σ , for $\tau=1$, $m=\mu=1$ and different δ

Figure 5 presents W for $z=0.6$, z_{\min} and H^* as functions of σ , for $\delta=0.5$, $m=\mu=1$ and different τ . The second and third panels are qualitatively similar to the second and third panels in Fig. 4. For the first panel W still increases in σ , as in Fig. 4, but the defender prefers large τ in Fig 5 in contrast to preferring low δ in Fig. 4, and conversely for the attacker.

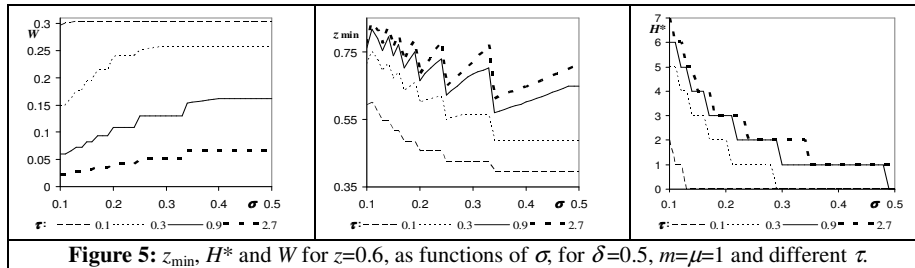


Figure 5: z_{\min} , H^* and W for $z=0.6$, as functions of σ , for $\delta=0.5$, $m=\mu=1$ and different τ

3. Attacker chooses a subset of Targets to attack

If the attacker survives the preventive strike, it observes $H+1$ possible targets and cannot distinguish the object and the FTs. However the attacker can decide to attack a randomly chosen subset of targets concentrating its resource in the attack against fewer FTs and hoping that the defended object is among the attacked targets. If the attacker attacks Q targets, $Q \leq H+1$, where Q is a free choice variable for the attacker, the per-target attack effort is $T=R/Q$ and the probability that the defended object is attacked is $Q/(H+1)$. The vulnerability of the object in the case when it is attacked is $v(H) = \frac{1}{1+(Q\tau)^\mu}$. The overall

probability of the object destruction in the case of the revenge attack is

$$G(Q) = \frac{Q}{(H+1)} \cdot \frac{1}{1+(Q\tau)^\mu}. \quad (8)$$

Considering the worst possible scenario for the defender, we assume that for any H the attacker can always choose or guess the value of Q that maximizes the probability of the object destruction:

$$Q^* = \arg \max_{0 \leq H \leq 1/\sigma} \left\{ G = \frac{Q}{(H+1)[1+(Q\tau)^\mu]} \right\}. \quad (9)$$

Differentiating with respect to Q we get

$$\frac{\partial G}{\partial Q} = \frac{1+(1-\mu)Q^\mu \tau^\mu}{(H+1)[1+(Q\tau)^\mu]^2} \quad (10)$$

If $\mu \leq 1$, $\frac{\partial G}{\partial Q} > 0$ and maximal destruction probability is achieved when $Q=H+1$. In this

case the attacker attacks all the targets and we have the situation considered in the previous section.

If $\mu > 1$, $\frac{\partial G}{\partial Q} = 0$ gives $Q^* = \frac{1}{\tau^\mu \sqrt[\mu]{\mu-1}}$ (for example, for $\mu=2$ $Q^*=1/\tau$). For the

convenience of later discussion, we denote $q = \left\lfloor \frac{1}{\tau^\mu \sqrt[\mu]{\mu-1}} \right\rfloor$. Since Q is integer and cannot

be greater than $H+1$, we have:

$$Q_H^* = \begin{cases} H+1 & \text{if } q \geq H+1 \\ q & \text{if } q < H+1, G(q) > G(q+1) \\ q+1 & \text{if } q < H, G(q) < G(q+1) \end{cases} \quad (11)$$

In the case of no preventive strike the probability of the destruction of the defended object \tilde{P} is

$$\tilde{P} = zG(Q_H^*, h) = \frac{zQ_H^*}{(h+1)[1+(Q_H^* \tau)^\mu]}. \quad (12)$$

In the case of preventive strike the probability of destruction of the defended object is

$$P(H) = (1-V(H))G(Q^*, H) = \frac{1}{1+((1-H\sigma)/\delta)^m} \cdot \frac{Q_H^*}{H+1} \cdot \frac{1}{1+(Q_H^* \tau)^\mu}. \quad (13)$$

The defender optimizes its resource distribution in order to minimize the probability of destruction of the defended object: $H^* = \arg \min_{0 \leq H \leq h} (P(H) \Rightarrow \min)$. Comparing (12) and (13), the preventive strike is justified if

$$P(H^*) < \tilde{P}, \text{ i.e., } \frac{1}{1 + ((1 - H^* \sigma) / \delta)^m} \cdot \frac{Q_{H^*}^*}{H^* + 1} \cdot \frac{1}{1 + (Q_{H^*}^* \tau)^\mu} < \frac{z Q_h^*}{(h+1) \left[1 + (Q_h^* \tau)^\mu \right]} \quad (14)$$

The defender should strike preventively if according to its estimates the probability of the attack against the defended object z exceeds the threshold value z_{\min} , where

$$z_{\min} = \frac{1}{1 + ((1 - H^* \sigma) / \delta)^m} \cdot \frac{h+1}{H^* + 1} \cdot \frac{Q_{H^*}^*}{Q_h^*} \cdot \frac{1 + (Q_h^* \tau)^\mu}{1 + (Q_{H^*}^* \tau)^\mu}. \quad (15)$$

The probability of object destruction given the optimal defense strategy is $W = \min\{\tilde{P}, P(H^*)\}$.

4. Illustrating the Solution of the Game

Figure 6 presents W for $z=0.7$, z_{\min} , H^* and Q^* as functions of σ , for $\delta=0.5$, $m=\tau=1$ and different μ . The destruction probability W increases in σ and decreases in the contest intensity μ in the attack against the defender. z_{\min} has an overall decreasing trend. Q^* decreases in σ since $Q^* \leq H^* + 1$. The attacker chooses $Q^* = H^* + 1$ when $\mu \leq 1$, and otherwise chooses $Q^* \leq H^* + 1$ since a large contest intensity is costly inducing the attacker to concentrate its resource into attacking few elements. When $\mu=2$, the attacker attacks only one element regardless of σ , despite many FTs being deployed.

Figure 7 presents W for $z=0.7$, z_{\min} , H^* and Q^* as functions of m , for $\delta=0.5$, $\sigma=0.18$, $\tau=1$ and different μ . Again W decreases in μ , but can be inverse U shaped (e.g., when $\mu=1$) in m . Hence when $\mu=1$ the attacker benefits from an intermediate m , while the defender benefits from m to be either low or high. With low m the defender enjoys the deployment of many FTs, whereas with high m fewer FTs make the defender benefit from concentrating its resource of protection.

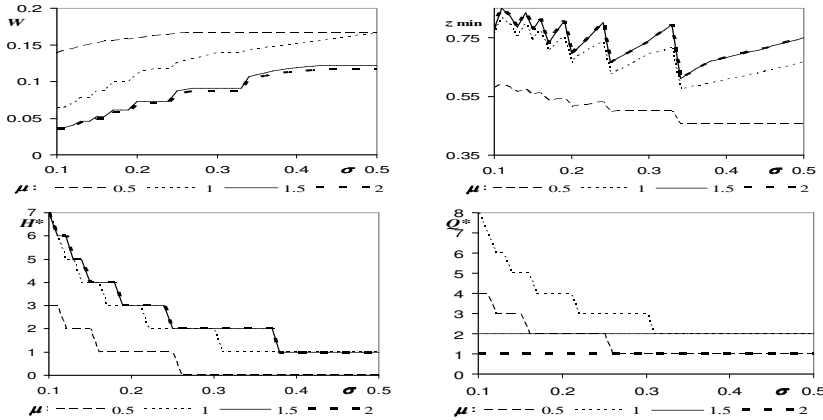


Figure 6: z_{\min} , H^* , Q^* and W for $z=0.7$, as functions of σ , for $\delta=0.5$, $m=\tau=1$ and different μ .

However, when both m and μ are low which makes the contest over each target more egalitarian in both contests, the attacker benefits despite the defender deploying many FTs. This latter result depends strongly on $\sigma=0.18$ which restricts the defender to deploy maximum five FTs. z_{\min} is inverse U shaped.

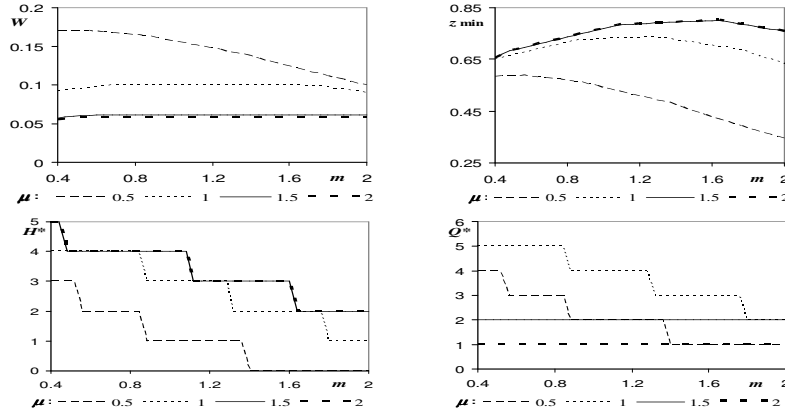


Figure 7: z_{\min} , H^* , Q^* and W for $z=0.7$, as functions of m , for $\delta=0.5$, $\sigma=0.18$, $\tau=1$ and different μ .

Figure 8 presents z_{\min} as function of m , for $\mu=1$, $\sigma=0.18$, $\tau=1$ and different δ (when $\tau=1$) and τ (when $\delta=0.5$). z_{\min} increases in δ and τ , and is increasing or inverse U shaped in m . When the attacker's protection effort is low compared with the total defender's resource, or the defender's protection effort is low compared with the attacker's attack resource, the defender justifies the preventive strike for a lower estimated attack probability. When the contest intensity m in the strike against the attacker is low, the defender justifies the preventive strike for a low estimated unprovoked attack probability, enjoying the more egalitarian contest. When δ is large (above 0.5 in Fig. 8) and τ is small ($\tau=1$ in Fig. 8), z_{\min} increases towards 1 when m increases, making the preventive strike unjustified even when the attacker is relatively certain to attack. The reason is that the defender is inferior in a double sense. Its protection effort is low and the attacker's protection effort is large. In this case the defender relies on deploying FTs instead, as we'll see in Fig. 9. For the remaining combinations of δ and τ , z_{\min} is inverse U shaped in m . For large m the defender justifies the preventive strike even when the attack probability is low. The reason is that the defender is no longer inferior in a double sense, and if the attacker is eliminated, its object is secure.

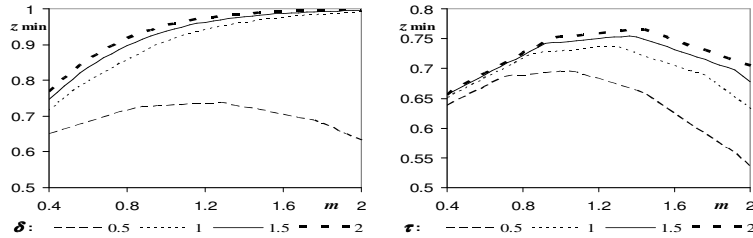


Figure 8: z_{\min} as function of m , for $\mu=1$, $\sigma=0.18$, $\tau=1$ and different δ (when $\tau=1$) and τ (when $\delta=0.5$).

Figure 9 presents z_{\min} , H^* , Q^* and W for $z=0.8$, as functions of τ , for $\sigma=0.18$, $m=\mu=1$ and different δ . It can be seen that W decreases in τ , causing decreasing destruction probability as the defender's protection effort increases relative to the attacker's attack resource. W is insensitive to δ when $\delta \geq 1$. The reason is that for $\delta \geq 1$, z_{\max} becomes greater than z and the defender prefers the passive defense and avoids the PS. In this case the attacker's protection resource has no influence on the outcome of its mission.

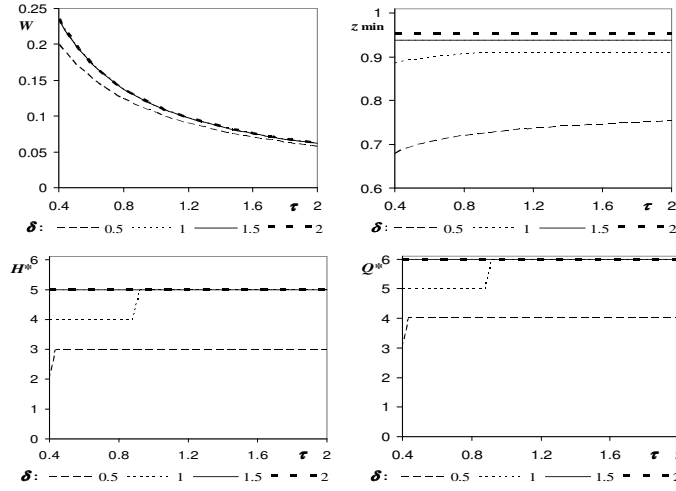


Figure 9: z_{\min} , H^* , Q^* and W for $z=0.8$, as functions of τ , for $\sigma=0.18$, $m=\mu=1$ and different δ .

z_{\min} increases in δ and becomes independent of τ for large δ . Indeed with the growth of τ and δ the defender benefits from increasing the number of FTs. At some point it allocates as many as possible FTs $H^*=h$ and (15) takes the form, $z_{\min} = \left[1 + ((1-h\sigma)/\delta)^m\right]^{-1}$. In this case z_{\min} does not depend on τ . The attacker attacks all targets because $Q^*=H^*+1$ is the solution of (9) for $\mu=1$.

5. Conservative Defense Strategy under Uncertain Contest Intensities

In many practical situations the values of the contest intensities cannot be exactly determined. Therefore, it would be useful to suggest a practical way to determine the optimal defense strategy for certain intervals of the contest intensities m and μ . The most conservative defense strategy is to assume that the actual values of m and μ (belonging to exogenously defined intervals) are the most favorable for the attacker. This approach is equivalent to assuming that the attacker can choose m and μ within the given interval as free strategic variables. The minmax defense strategy, thus, minimizes the maximal probability of the object destruction W associated with a combination of the most unfavorable circumstances (contest intensities m and μ) and the most harmful attacker's choice of Q .

Let $Q_H^*(m, \mu)$ be the value of the attacker's effort distribution parameter that maximizes the object destruction probability for the given H , m and μ . The defender's strategy is to choose the number of FTs H^* that minimizes W in the range $m_{\min} \leq m \leq m_{\max}$, $\mu_{\min} \leq \mu \leq \mu_{\max}$ of contest intensities assuming that the attacker always chooses its best response $Q_H^*(m, \mu)$:

$$\max_{\substack{m_{\min} \leq m \leq m_{\max} \\ \mu_{\min} \leq \mu \leq \mu_{\max}}} W[H^*, Q_H^*(m, \mu), m, \mu] \leq \max_{\substack{m_{\min} \leq m \leq m_{\max} \\ \mu_{\min} \leq \mu \leq \mu_{\max}}} W[H, Q_H^*(m, \mu), m, \mu] \quad (16)$$

for any $H \neq H^*$.

In order to solve the minmax game for any given ranges of the contest intensities the following procedure should be applied:

1. Find $1 \leq Q_H^* < h+1$, and $\mu_{\min} \leq \mu^* \leq \mu_{\max}$ that maximize \tilde{P} ;
2. Assign $P_{\min}=1$;
3. For each $H=0, \dots, h$
 - 3.1. Find $1 \leq Q_H^* < H+1$, $m_{\min} \leq m^* \leq m_{\max}$ and $\mu_{\min} \leq \mu^* \leq \mu_{\max}$ that maximize $P(Q, H)$;
 - 3.2. If $P(H) < P_{\max}$ assign $P_{\max}=P(Q_H^*, H)$, $H^*=H$;
4. If $P_{\max} < \tilde{P}$, allocate H^* FTs and strike preventively achieving $W=P_{\max}$, otherwise allocate h FTs and abstain from striking preventively achieving $W=\tilde{P}$.
5. Obtain $z_{\min} = \tilde{P} / (zP_{\max})$.

Figure 10 presents z_{\min} , m^* , μ^* , H^* , Q_H^* and W for $z=0.7$, as functions of τ and δ for $\sigma=0.18$ and uncertain contest intensities that can take values in the ranges $1 \leq m \leq 3$, $0 \leq \mu \leq 2$. Fig. 11 presents the same functions for the case of two times shorter ranges of the uncertain contest intensities (with the same mean values): $1.5 \leq m \leq 2.5$, $0.5 \leq \mu \leq 1.5$. One can see that the most harmful for the defender values of m and μ can abruptly change with variation of τ and δ . Indeed, when the game parameters change, the defender and the attacker adjust their H and Q_H accordingly, which causes variation of the effort balances in the attack and the preventive strike. When $(1-H^*\sigma) < \delta$ in (14), the defender suffers from the maximal possible value of m , otherwise the defender suffers from the minimal possible value of m . When $Q_H^* \tau < 1$ in (14), the minimal possible value of μ is most favorable for the defender, otherwise the maximal possible value of μ is most favorable for the defender.

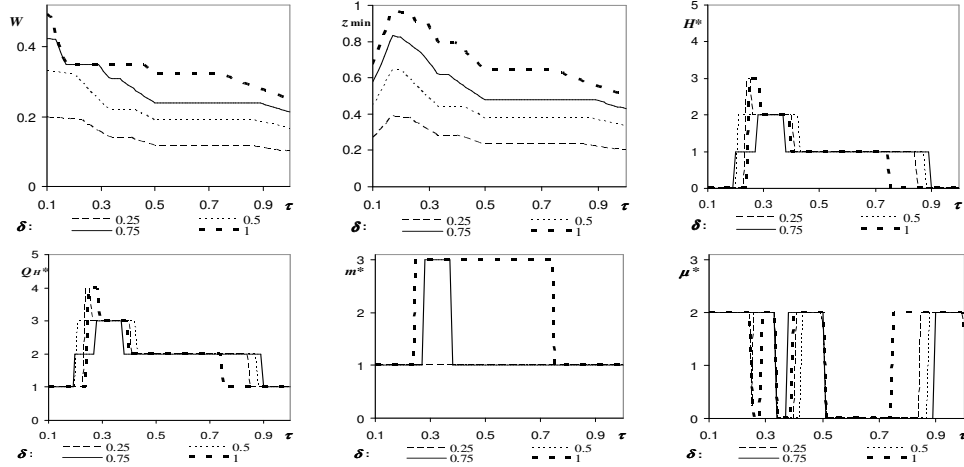


Figure 10: z_{\min} , m^* , μ^* , H^* , Q_H^* and W for $z=0.7$, as functions of τ and δ for $\sigma=0.18$, for $1 \leq m \leq 3$, $0 \leq \mu \leq 2$.

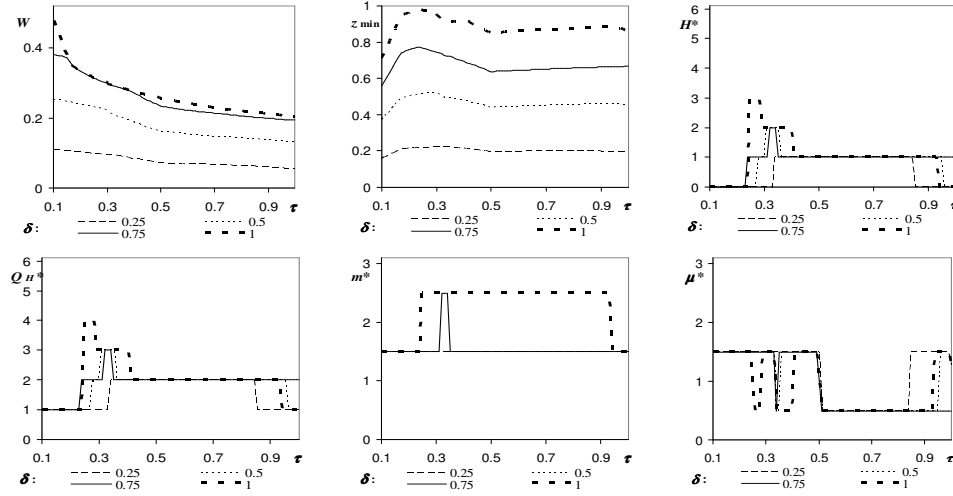


Figure 11: z_{\min} , m^* , μ^* , H^* , Q_H^* and W for $z=0.7$, as functions of τ and δ for $\sigma=0.18$, for $1.5 \leq m \leq 2.5$, $0.5 \leq \mu \leq 1.5$.

Comparing Fig. 10 with Fig. 11 one can see that the defender benefits from the reduction of the uncertainty of the contest intensities. Indeed, when $1.5 \leq m \leq 2.5$, $0.5 \leq \mu \leq 1.5$, W is always lower than when $1 \leq m \leq 3$, $0 \leq \mu \leq 2$.

6. Conclusion

The article analyzes how a defender determines a balance between defending an object passively by deploying false targets (decoys) and striking preventively against an attacker seeking to destroy the object. With no preventive strike the defender allocates its entire resource to passive protection. If the defender strikes preventively, the attacker's vulnerability depends on its protection and on the defender's resource allocated to the strike. If the attacker survives, the object's vulnerability depends on the attacker's revenge attack resource allocated to the attacked object. The attacker cannot distinguish the false targets from the genuine object and can choose to attack a subset of the targets.

The paper presents an optimization model that can be used for making a decision about efficiency of the preventive strike based on the estimated attack probability. The methodology of analysis of influence of different model parameters on the optimal defense strategy is demonstrated. It is shown that the preventive strike is beneficial for the defender when the deployment of FTs becomes expensive, when the probability of unprovoked attacker's strike is high and when the attacker's protection effort is low compared with the total defender's resource, or the defender's protection effort is low compared with the attacker's attack resource.

It is also shown that the contest intensity parameters m and μ , for the attacker's vulnerability and the object's vulnerability, respectively, strongly influence the optimal defense strategy. The minimal estimated attacker's strike probability when the preventive strike is justified can depend on the contest intensity parameters non-monotonically, which complicates the analysis and makes intuition based decision making impossible.

The contest intensity parameters usually cannot be exactly evaluated in practice. We demonstrate the most conservative approach of handling the uncertainty of the contest intensities in which the range of possible variation of m and μ is determined and the

"worst case" defense strategy is obtained under the assumption that m and μ take the values that are most favorable for the attacker (in this case m and μ can be considered as additional strategic variables that the attacker can choose within the specified ranges).

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