

## Production Rate Maximization of a Multi-State System under Inspection and Repair Policy

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**Abstract:** In this paper, an inspection strategy for a multi-state system is proposed. This system can be in any of three states: nominal operating state, degraded state or failure state. The system state is known only after inspection. A maintenance action is undertaken when at a predetermined instant an inspection reveals that the system is in degraded or failure state. The maintenance action restores the system to its nominal operating mode with a certain probability. For this study, a periodic type inspection strategy is used. It aims at maximizing the productivity of the considered multi-state system. Analytical and numerical results are presented.

**Keywords:** *Optimization, Production rate, Inspection, Multi-state system*

### 1. Introduction

Technological systems are characterized more and more by the permanent complexity of their structure. In reliability engineering, the operation of these systems is usually modeled by a binary approach where only two states are permitted: nominal functioning, and complete failure. However, in many real-life situations, this binary-state assumption may not be adequate. In multi-state reliability modeling, the system may rather have more than two levels of performance varying from perfect functioning to complete failure. The presence of degradation is a common situation in which a system should be considered to be multi-state. Degradation can be caused by system deterioration or by variable ambient conditions. Fatigue, failures of non-essential components, and number of random shocks on the system are all examples of system degradation causes. In this case, the failure rate depends on the status of the system which can degrade gradually. The reliability analysis of such degraded systems should consider multiple operational states to take into account multiple degradation levels [22, 32]. In the literature, many papers study unit, series, parallel, bridge or  $k$ -out-of- $n$  multi-state configurations [16, 25]. A literature review on multi-state systems (MSS) reliability can be found for example in [22].

Maintenance plays a key role to improve the performance of a multi-state degraded system. Maintenance concepts assume that perfect repair brings a system to the "good as new" state and minimal repair brings the system to the "bad as old" state (repair gets the system back into operation without changing its failure rate).

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However, most deteriorating systems after maintenance will be restored to a condition between "good as new" state and "bad as old" state. That is, after repair, a deteriorating system will be restored and/or retained to a condition which is better than before failure or maintenance, but worse than the brand new one. This type of maintenance model is called the imperfect maintenance [26]. A well-known model in which upon repair a system will be "good as new" with probability  $p$  and "bad as old" with probability  $q = 1-p$  is discussed in [9] for a single-unit system. In [27], the authors studied a series system of  $n$  components subjected to imperfect repair. The later is modeled in a way that upon it, the time to failure of a component will decrease to a fraction of its immediately previous one, and the repair time will increase to a multiple of its immediately previous one. Inspections are an important part of a maintenance program because they provide a way to discover dormant failures and/or degradation that is likely to result in the imminent failure of a component or a system. Maintenance actions such as inspections, repairs or replacements are taken to avoid operating the system in an undesirable state. But, these actions may interrupt production and increase downtime and total maintenance costs [29]. In [17, 18], the author studied an availability Markov model of a multi-state system with an optimal inspection policy by introducing a profit maximization model under inspection policy. Markov processes and semi-Markov processes can be used for modeling the transition of those states [28]. State transitions occur according to a Poisson process. For this process, a periodic inspection strategy can be optimal [6, 10]. The use of partially observable Markov decision process structure to select maintenance actions that minimize the total expected cost for operating a system has been studied in [29]. This cost includes the cost per unit time for operating the system in each state as well as the cost of performing actions to change the system state.

A system that allows service continuity under failure with a reduced level of functionality [15, 3, 4, 1, 22] is multi-state. Several technological strategies are proposed in the literature to allow a system to continue its operation with a reduced performance. These strategies include reconfiguration of the control system, physical reorganization of the system components, robust supervisory-control design, redundancy, and intercellular transfers for cellular manufacturing systems [11, 24, 31]. The degraded state can result from the activation of automatic reconfiguration mechanisms (material and/or software). This allows the system to ensure the continuity of its service despite a failure of one or several of its non-essential components. In this case, the failure involves a performance degradation of the system, but not its complete failure. In this article, we consider a production system that can operate either in a nominal mode or in a degraded mode. This system can also fail randomly. The system state is known only after inspection. A maintenance action is undertaken when at a predetermined instant an inspection reveals that the system is in degraded or failure state. The maintenance action restores the system to its nominal operating mode with a certain probability. A periodic type inspection strategy is used, and the objective is to maximize the productivity of the system. This is an important problem that has not been, to the best of our knowledge, studied in the literature. In Section 2, the optimization problem is formulated. A numerical example is presented in Section 3, and Section 4 concludes the paper.

## 2. Optimization Problem Formulation

One of the main objectives of an effective maintenance system is to maximize the availability of equipment. Availability is the ability of an item to perform its required function at a given time or over a stated period of time when operated and maintained in a prescribed manner [14]. Increasing the availability of the equipment increases its productivity, and hopefully its profitability [19]. This assertion is true for binary systems, but is not always verifiable in the multi-state case. The considered MSS is profitable when it is less at its degraded state. In sum, it is necessary that the proportion of time spent at the degraded state is lower than its nominal state. Let  $P_N(t)$  represents the probability that the system operates in nominal state at time  $t$  with a production rate  $TP_N$ , and let  $P_D(t)$  the probability that it operates in degraded state at time  $t$  with a production rate  $TP_D$  ( $TP_N > TP_D$ ). The availability  $A(t)$  and the overall production rate  $TP_G$  are:

$$A(t) = P_N(t) + P_D(t), \quad (1)$$

$$TP_G = TP_N P_N(t) + TP_D P_D(t) \quad (2)$$

The transition diagram of Figure 1 illustrates the different states of the studied system.

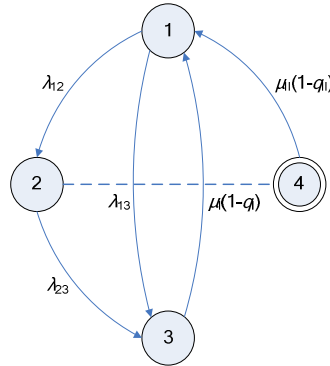


Figure 1: System Transitions Diagram

### Notation

State 1: Nominal operating state.

State 2: Degraded state.

State 3: Failure state.

State 4: The inspection revealed that the system operates in a degraded mode. A maintenance action is immediately undertaken. State 4 is a fictitious state.

$x$ : Time interval between two inspections.

$\lambda_{ij}$ : Failure rate from state  $i$  to state  $j$ .

$P_i(t)$ : Probability that the system is in state  $i$  at time  $t$ ,  $i=1, \dots, 4$ , with  $0 \leq t \leq x$

$(\sum_{i=1}^4 P_i(t) = 1)$ .

Repair I: Repair action after total failure.

Repair II: Repair action when the system operates in degraded mode.

$\mu_I, \mu_{II}$ : Repair rate (constant) for repair actions of type I and II.

$q_i$ : Probability that repair I is carried out in an imperfect way.

$q_{II}$ : Probability that repair II is carried out in an imperfect way.

### Assumptions

1. Nominal and degraded states are known only after inspection.
2. If the system has completely failed, a maintenance action is immediately undertaken to bring it back to nominal operating state. In this case, repair times are independent and identically distributed according to an exponential distribution with average  $1/\mu_I$ .
3. If the inspection reveals that the system operates in degraded state, a maintenance action is immediately undertaken to bring back it to nominal operating state. In this case, repair times are independent and identically distributed according to an exponential distribution with average  $1/\mu_{II}$ .
4. A conditional probability  $(1-q_I)$  is associated to repair I if the system is successfully brought back from failure state (state 3) to nominal operating state (state 1). Probability  $q_I$  is assumed to be known.
5. A conditional probability  $(1-q_{II})$  is associated to repair II if the system is successfully brought back from degraded state (state 4) to nominal operating state (state 1). Probability  $q_{II}$  is assumed to be known.
6. Production rates in nominal mode ( $TP_N$ ) and degraded mode ( $TP_D$ ) are known and constant (expressed in number of parts by time unit).
7. Inspection is carried out in a perfect way and does not affect the system production rate. This inspection accurately reveals the system state.

The optimization problem is to determine the inspection periodicity  $x$  which maximizes the overall production rate given by equation (2), with  $P_N(t)$  and  $P_D(t)$  representing, respectively, the probabilities of state 1 and 2. These probabilities  $P_1(t)$  and  $P_2(t)$  are obtained by solving the following system of differential equations:

$$\frac{dP_1(t)}{dt} = -(\lambda_{12} + \lambda_{13})P_1(t) + \mu_I(1 - q_I)P_3(t) + \mu_{II}(1 - q_{II})P_4(t), \quad (3)$$

$$\frac{dP_2(t)}{dt} = -\lambda_{23}P_2(t) + \lambda_{12}P_1(t), \quad (4)$$

$$\frac{dP_3(t)}{dt} = -\mu_I(1 - q_I)P_3(t) + \lambda_{13}P_1(t) + \lambda_{23}P_2(t), \quad (5)$$

$$\frac{dP_4(t)}{dt} = -\mu_{II}(1 - q_{II})P_4(t). \quad (6)$$

According to the inspection strategy in [17, 18], we define the initial conditions of this system of differential equations:

$$P_1(0)=P_1(x), \quad P_2(0)=0, \quad P_3(0)=P_3(x), \quad P_4(0) = P_2(x)+P_4(x). \quad (7)$$

Knowing these initial conditions, the expressions of  $P_i(t)$  are obtained after taking Laplace transforms of equations (3)-(6). The reader is referred to [2] for more details about this calculation. The inspection periodicity that maximizes the production rate of the system is obtained as follows. The overall production rate ( $TP_G$ ) of the system is given by:

$$TP_G = TP_N P_1(x) + TP_D P_2(x). \quad (8)$$

The studied optimisation model is:

$$\text{Maximize } Z = TP_N P_1(x) + TP_D P_2(x), \quad (9)$$

$$\text{Subject to } K(x) \leq B, \quad (10)$$

where  $K(x)$  is the incurred average total cost per unit of time when the inspection periodicity  $x$  is applied.  $B$  is the budget per unit of time allocated to maintenance (repair cost) and penalties due to production lost. The expression of  $K(x)$  is:

$$K(x) = C_3 z_3(x) + C_4 z_4(x) + \frac{I_c}{x} (1 - P_3(x) - P_4(x)), \quad (11)$$

where,

$$z_i = \frac{1}{x} \int_0^x P_i(t) dt, \quad i = 3, 4, \quad (12)$$

$C_i$  is the average repair cost and loss per unit of time when the system is in state 3 or state 4 ( $C_3 > C_4$ ), and  $I_c$  is the inspection cost. Here,  $z_3$  and  $z_4$  represent the average proportions of time that the system spends in states 3 and 4 (respectively) during the interval  $[0, x]$ . The optimization problem given by equations (9) and (10) will be solved numerically for the example considered in the next section.

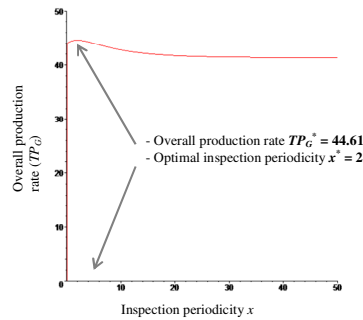
### 3. An Illustrative Example

The system parameters are given in Table 1. The optimization problem is solved numerically using *Maple* software. The obtained optimal inspection periodicity ( $x^*$ ) is 2 units of time. It involves an overall production rate  $TP_G^*$  of 44.61 items per time unit. For this optimal value of  $x$ , we have  $P_1(0)=0.8299$ ,  $P_2(0)=0$ ,  $P_3(0)=0.0785$ ,  $P_4(0)=0.0914$ ,  $P_1(x^*)=0.8299$ ,  $P_2(x^*)=0.0777$ ,  $P_3(x^*)=0.0785$ , and  $P_4(x^*)=0.0136$ . Figures 2 and 3 illustrate, respectively, the objective function  $TP_G$  and the average total cost per time unit  $K(x)$ .

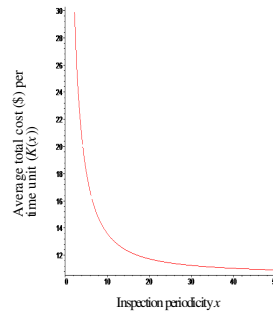
**Table 1:** Data of System Characteristic Parameters

$\lambda_{12}$	$\lambda_{13}$	$\lambda_{23}$	$\mu_I$	$\mu_{II}$	$q_I$	$q_{II}$	$TP_N$	$TP_D$	$C_3$	$C_4$	$I_c$	$B$
0.05	0.04	0.08	0.5	1	0.05	0.05	50	40	100	75	50	100

The larger the inspection frequency ( $1/x$ ) is, the more the system will operate in nominal mode. Inversely, if the inspection frequency is low, the system is likely to operate longer in degraded mode. This eventuality reduces the overall production rate ( $TP_G$ ) as shown by Figure 2. Figure 3 illustrates how that  $K(x)$  decreases when  $x$  increases.



**Figure 2:** Evolution of  $TP_G$  according to Inspection Periodicity  $x$



**Figure 3:** Shape of  $K(x)$  according to Inspection Periodicity  $x$

#### 4. Conclusion

In this study, an inspection strategy was proposed. The production system considered was subjected to random failures. This system can operate in nominal or degraded states. These states are known only after inspection, and inspections are carried out at predetermined time intervals. The complete failure of the system is detected instantly. Maintenance actions bringing the system to nominal operating state are imperfectly executed. The objective is to determine the inspection periodicity that maximizes the overall production rate of the system for a specific situation. The repair and failure rates are assumed to be constant. The obtained results are coherent, and they show that the overall production rate grows with the frequency of inspections. In addition, the average total cost increases with the probability that system stayed in an undesirable state. The proposed model can be applied for many practical situations where it is important to take into account the degraded functioning mode. Future work will extend this model to deal with multi-component degraded multi-state systems.

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