

A Novel Importance Measure for External Factors Based on System Performance

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Abstract: Importance measures and analysis have been used to identify weak components to prioritize system upgrading activities, maintenance activities, *etc.* Traditionally, importance measures do not consider the possible effect due to external environment and phenomena, which however can be causes of system failures and therefore should be taken into consideration. This paper proposes a novel importance measure for multi-state systems with the consideration of external factors. And the proposed importance analysis can effectively quantify the effect of the state of the external factor on the component and system performance.

Keywords: *External factor, importance measure, performance, multi-state system*

1. Introduction

Common causes and their prevention have been a serious concern in reliability and safety community. Examples of common cause mechanisms include design deficiency, external environment, external phenomena, functional deficiency, and human factors [1]. Among them, the design deficiency, functional deficiency and human factors can be prevented by quality assurance, testing and human communication. The external environment can be from contamination of fluid systems, corrosion, and electrical noise such as dirt, dust, humidity and temperature. The external phenomena can be categorized into two classes. The first class is from natural disasters such as earthquakes, fires, floods, storms, and temperature extremes. The second class is from falling objects, flywheel missiles, pipe-whip, small steam leaks, and vibration. In this paper, the external environment and phenomena are referred to as external factors, which are highly unpredictable and uncontrollable. It is very important to measure the importance of external factors, which is useful to investigate external factors' effects or contributions to the system function or performance.

Importance measures have been widely used for identifying the weakest component and supporting system improvement activities for system reliability and performance. Importance measures were first introduced by Birnbaum [2] in 1969. The Birnbaum importance measure gives the contribution of the component reliability to the system reliability. Based on the Birbaum's measure, Fussell-Vesely [3, 4], Vaurio [5] *et al.* have improved the theory and applications of importance measures since 1970s. Traditionally, importance measures do not consider the possible effect due to external factors, which however can be causes of system failures and therefore should be taken into consideration. In this paper, we study the mechanism of the effect of the external factors on the system reliability and performance through the internal components. Firstly, we analyze the change of the probability of component state caused by the appearance of external factor. Secondly, we give a novel importance measure to describe how the external factors affect the system performance, to rank different external factors, and to provide guidance for

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improving the design of system and component.

2. Proposed Model

Assume that the set of external factors is $\{ec_1, ec_2, \dots, ec_M\}$, $x_{ec_k} \in \{0, 1, 2, \dots, s_k\}$ represents the state of external factor ec_k , where $x_{ec_k} = 0$ represents that external factor ec_k does not appear and $x_{ec_k} = 1, 2, \dots, s_k$ represent that the external factor ec_k appears with different states or performance levels. Based on the total probability law, the probability that component i is at state m is computed as follows.

$$P_{im} = P(x_i = m) = \sum_{b=0}^{s_k} P(x_{ec_k} = b)P(x_i = m | x_{ec_k} = b), \quad (1)$$

where x_i represents the state of component i , the state space of component i is $\{0, 1, 2, \dots, M_i\}$, $\sum_{m=0}^{M_i} P_{im} = 1$ and $\sum_{b=0}^{s_k} P(x_{ec_k} = b) = 1$.

Consider component i being at state m . When external factor ec_k appears and is at state l ($l > 0$), the conditional probability that component i is at state m is $P(x_i = m | x_{ec_k} = l)$. So the absolute change of the probability that component i is at state m is

$$\begin{aligned} & |P(x_i = m | x_{ec_k} = l) - P(x_i = m)| \\ &= |P(x_i = m | x_{ec_k} = l) - \sum_{b=0}^{s_k} P(x_{ec_k} = b)P(x_i = m | x_{ec_k} = b)| \\ &= |P(x_i = m | x_{ec_k} = l) - P(x_{ec_k} = l)P(x_i = m | x_{ec_k} = l) - \sum_{b=0, b \neq l}^{s_k} P(x_{ec_k} = b)P(x_i = m | x_{ec_k} = b)| \\ &= \left| \sum_{b=0, b \neq l}^{s_k} P(x_{ec_k} = b)P(x_i = m | x_{ec_k} = l) - \sum_{b=0, b \neq l}^{s_k} P(x_{ec_k} = b)P(x_i = m | x_{ec_k} = b) \right| \\ &= \left| \sum_{b=0, b \neq l}^{s_k} P(x_{ec_k} = b) [P(x_i = m | x_{ec_k} = l) - P(x_i = m | x_{ec_k} = b)] \right| \\ &= \sum_{b=0}^{s_k} P(x_{ec_k} = b) [P(x_i = m | x_{ec_k} = l) - P(x_i = m | x_{ec_k} = b)] l, \end{aligned} \quad (2)$$

and the expected absolute change of the probability that component i is at state m is

$$\begin{aligned} & E(|P(x_i = m | x_{ec_k} = l) - P(x_i = m)|) \\ &= \sum_{l=1}^{s_k} P(x_{ec_k} = l) \sum_{b=0}^{s_k} P(x_{ec_k} = b) [P(x_i = m | x_{ec_k} = l) - P(x_i = m | x_{ec_k} = b)] l. \end{aligned} \quad (3)$$

Equation (2) is the absolute deviation between $P(x_i = m | x_{ec_k} = l)$ and $P(x_i = m)$. It measures the absolute change of the probability that component i is at state m caused by the appearance of a certain external factor's state $ec_k = l$. Equation (3) is the expected absolute deviation between $P(x_i = m | x_{ec_k} = l)$ and $P(x_i = m)$. It measures the expected absolute change of the probability that component i is at state m caused by the appearance of a particular external factor's different state levels, and associated probabilities.

3. Importance Measure of External Factors

Assume the state space of the system is $\{0, 1, \dots, M\}$, where 0 represents the complete failure of system and M is the perfect functioning state of system. The states are ordered from 0 to M . When considering the effect of external factors on the system performance,

based on [6], we have

$$U = \sum_{j=1}^M (a_j - a_{j-1}) P[\Phi(0, X) \geq j] + \left(\frac{\partial U}{\partial \rho_{i1}}, \frac{\partial U}{\partial \rho_{i2}}, \dots, \frac{\partial U}{\partial \rho_{iM_i}} \right) \cdot \rho_i^T, \rho_i = (\rho_{i1}, \rho_{i2}, \dots, \rho_{iM_i}), \quad (4)$$

where U represents the expected performance of a system, a_j represents the performance level corresponding to state j of the system ($0 = a_0 \leq a_1 \leq \dots \leq a_M$), $\Phi(X)$ represents the system structure function, $X = (x_1, x_2, \dots, x_n)$ represents the state vector of the components, $\rho_{im} = P_{im} + P_{i(m+1)} + \dots + P_{iM_i}$ and $\frac{\partial U}{\partial \rho_{im}} = \sum_{j=1}^M (a_j - a_{j-1}) [P(\Phi(m, X) \geq j) - P(\Phi((m-1), X) \geq j)]$.

According to (1), we have

$$\rho_{im} = \sum_{c=m}^{M_i} \left[\sum_{b=0}^{s_k} P(x_{ec_k} = b) P(x_i = c \mid x_{ec_k} = b) \right] = \sum_{b=0}^{s_k} P(x_{ec_k} = b) \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = b). \quad (5)$$

So the latter part of Equation (4) can be converted into

$$\begin{aligned} & \left(\frac{\partial U}{\partial \rho_{i1}}, \frac{\partial U}{\partial \rho_{i2}}, \dots, \frac{\partial U}{\partial \rho_{iM_i}} \right) \cdot \rho_i^T = \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \rho_{im} \\ &= \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \left[\sum_{b=0}^{s_k} P(x_{ec_k} = b) \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = b) \right] \\ &= \sum_{b=0}^{s_k} P(x_{ec_k} = b) \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = b). \end{aligned} \quad (6)$$

where $P(x_{ec_k} = 0) = 1 - \sum_{d=1}^{s_k} P(x_{ec_k} = d)$.

Let the vector $(P(x_{ec_k} = 1), P(x_{ec_k} = 2), \dots, P(x_{ec_k} = s_k))$ describe the distribution of x_{ec_k} . According to (4), (5) and (6), we have

$$\begin{aligned} \frac{\partial U}{\partial P(x_{ec_k} = j)} &= \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = j) - \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = 0), j \geq 1 \\ &= \sum_{m=1}^{M_i} \frac{\partial U}{\partial \rho_{im}} \cdot \left[\sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = j) - \sum_{c=m}^{M_i} P(x_i = c \mid x_{ec_k} = 0) \right]. \end{aligned} \quad (7)$$

Equation (7) describes the effect of the state of the external factor on the system performance. With the importance values, proper actions can be taken on the weakest external factor to improve system performance at the minimum effort.

4. Case Study

In this section, we study a single component system subject to a single external factor. Assume the state vector of the external factor is $(0, 1)$, representing non-occurrence and occurrence of the external factor, respectively. The state space of the component is $\{0, 1, 2\}$. Then the system expected performance can be expressed as

$$\begin{aligned} U &= a_0 P(x_1 = 0) + a_1 P(x_1 = 1) + a_2 P(x_1 = 2) \\ &= a_0 [P(x_{ec_1} = 0) P(x_1 = 0 \mid x_{ec_1} = 0) + P(x_{ec_1} = 1) P(x_1 = 0 \mid x_{ec_1} = 1)] \\ &\quad + a_1 [P(x_{ec_1} = 0) P(x_1 = 1 \mid x_{ec_1} = 0) + P(x_{ec_1} = 1) P(x_1 = 1 \mid x_{ec_1} = 1)] \\ &\quad + a_2 [P(x_{ec_1} = 0) P(x_1 = 2 \mid x_{ec_1} = 0) + P(x_{ec_1} = 1) P(x_1 = 2 \mid x_{ec_1} = 1)], \end{aligned}$$

so we have

$$\begin{aligned}
& \frac{\partial U}{\partial P(x_{ec_1}=1)} \\
&= a_0 [P(x_1=0|x_{ec_1}=1) - P(x_1=0|x_{ec_1}=0)] + a_1 [P(x_1=1|x_{ec_1}=1) - P(x_1=1|x_{ec_1}=0)] \\
&+ a_2 [P(x_1=2|x_{ec_1}=1) - P(x_1=2|x_{ec_1}=0)] \\
&= -a_0 \left[\sum_{c=1}^2 P(x_1=c|x_{ec_1}=1) - \sum_{c=1}^2 P(x_1=c|x_{ec_1}=0) \right] + a_1 \left[\sum_{c=1}^2 P(x_1=c|x_{ec_1}=1) - \sum_{c=1}^2 P(x_1=c|x_{ec_1}=0) \right] \\
&+ (a_2 - a_1) [P(x_1=2|x_{ec_1}=1) - P(x_1=2|x_{ec_1}=0)] \\
&= (a_1 - a_0) \left[\sum_{c=1}^2 P(x_1=c|x_{ec_1}=1) - \sum_{c=1}^2 P(x_1=c|x_{ec_1}=0) \right] + (a_2 - a_1) [P(x_1=2|x_{ec_1}=1) - P(x_1=2|x_{ec_1}=0)].
\end{aligned}$$

The expression of U can be converted into

$$\begin{aligned}
U &= a_0 P(x_1=0) + a_1 P(x_1=1) + a_2 P(x_1=2) \\
&= a_0 (1 - P(x_1=1) - P(x_1=2)) + a_1 (P(x_1=1) + P(x_1=2)) + (a_2 - a_1) P(x_1=2) \\
&= a_0 (1 - \rho_{11}) + a_1 \rho_{11} + (a_2 - a_1) \rho_{12} \\
&= a_0 + (a_1 - a_0) \rho_{11} + (a_2 - a_1) \rho_{12},
\end{aligned}$$

$$\text{Thus, } \frac{\partial U}{\partial \rho_{11}} = a_1 - a_0, \frac{\partial U}{\partial \rho_{12}} = a_2 - a_1.$$

$$\text{Hence, } \frac{\partial U}{\partial P(x_{ec_1}=1)} = \sum_{m=1}^2 \frac{\partial U}{\partial \rho_{1m}} \cdot \left[\sum_{c=m}^2 P(x_1=c|x_{ec_1}=1) - \sum_{c=m}^2 P(x_1=c|x_{ec_1}=0) \right], \text{ which corresponds to (7).}$$

5. Conclusions

In this paper, we have studied the effect of external factors on the system performance. A novel definition of importance measure for external factors has been presented. The proposed importance measure describes the effect of the state of the external factor on the system performance. With the importance values, proper actions can be taken on the weakest external factor to improve the system performance at the minimum effort. A case study has been performed to demonstrate the process to calculate the importance measure of external factors.

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