

Reliability Model of Tracking, Telemetry, Command and Communication System using Markov Approach

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Abstract: For the reliability analysis of tracking, telemetry and command (TT&C) and communication systems, most existing modeling methods can only deal with general TT&C and communication tasks. In this paper, a formal description of TT&C and communication task is given to facilitate the reliability modeling of such systems. A continuous-time Markov chain (CTMC) model is built for an idle task arc. A model for TT&C and communication tasks in consecutive flight cycles is proposed, in which the tasks are combined to a new complicated one. Examples with numerical results show the effectiveness of the proposed approach.

Keywords: *TT&C and Communication system, reliability model, Markov process.*

1. Introduction

The TT&C and communication system plays very important roles in space flight activities. The main tasks of it are tracking, surveying, monitoring and controlling the spacecraft [1], and communication, such as voice, television and data transmission [2]. Therefore, its system reliability is essential to the success of launching, motioning and landing of spacecraft.

Various techniques have been developed for the reliability analysis of TT&C and communication systems: Wolman [3] constructed a reliability model of space systems which is defined in a set-theoretic framework; Bondavalli *et al.* [4] proposed a modeling approach based on a hierarchical methodology to solve the Rosetta space application provided by Matra Marconi Space. Essentially, the TT&C and communication task can be regarded as a phased-mission system (PMS). For example, the task has diverse types and multi-phase. PMS has been widely studied in literature [5-8]. The modeling and analyzing methods in literature may be roughly classified into two major groups: combinatorial methods and Markov methods. Combinatorial methods include reliability block diagram (RBD) [5, 9] and fault tree analysis [10, 11]. These methods are straightforward, efficient, but limited to handle non-repairable system. However, the methods based on Markov process can correctly treat dependency between different components and two successive phases.

Generally, the TT&C and communication task is not an ordinary kind of PMS. For example, it includes phases referred to as idle task arcs, in which no task is implemented. In this paper, by defining concepts and notations of the TT&C and communication system, models are built for idle task arcs and tasks in consecutive flight cycles.

2. Formal Descriptions of TT&C and Communication Task

Some relevant concepts and notations of the TT&C and communication system are given as follows.

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Flight Phase: the spacecraft flight process can be divided into several phases. Let fp^i represent the i th flight phase.

Flight Cycle: the period of spacecraft orbiting around the earth is called a flight cycle. The TT&C and communication system should perform various tasks during each flight cycle to support space mission. Let $fc^{j,*}$ represent the j th flight cycle, in which $*$ = *key* is the critical flight cycle, and $*$ = *Nonkey* is the non-critical one.

Resources Set of a Task: a task needs a number of TT&C and communication resources to implement it. Resources include stations, centers, ships, mobile stations, and facilities or sub-systems. If a resource has the ability to implement TT&C and communication task, it must “see” the spacecraft. Due to the restriction of orbit visibility, a resource can “see” a spacecraft only within some special time periods, and different resources have different visible time periods. It means that for the same task, different resources may have different implementing time periods. The resource set of a task is denoted as *Resources*, element in *Resources* is defined as $res_k = \{rname, rst, ret\}$, where, *rname* is the name of resource, *rst* and *ret* are the start time and the end time respectively for implementing task. For example, $Resources = \{(r_1, t_1, t_2), (r_2, t_3, t_4)\}$ means resource r_1 and r_2 implement a task, and the start time and end time are t_1, t_2 , and t_3, t_4 respectively.

TT&C and Communication Task (TTCCT): tasks implemented by the TT&C and communication system have multiple types, and they are performed during a certain flight cycle of a flight phase. The TTCCT can be defined as $task_i = \{type, fp^i, fc^{j,*}, tst, tet, Resources\}$, where, *type* is the type of $task_i$, fp^i represents that $task_i$ is performed during the i th flight phase, $fc^{j,*}$ represents that $task_i$ is performed during the j th flight cycle, *tst* and *tet* are the start time and the end time of $task_i$ respectively, and *Resources* is the resource set of $task_i$.

Task Arc: as discussed above, resources in a task may have different implementing time periods. Therefore, the system configuration and success criterion of TTCCT may change over time. In order to analyze the reliability of TTCCT, it should be transformed into a sequence of single phase systems. Each single phase system is called a task arc, and the m th task arc denoted as arc_m . Let $Arcs(task_i)$ be the set of all task arcs in $task_i$. The system topology in each arc can be expressed by a RBD or fault tree.

Idle Task Arc: a specific task arc, in which all resources are idle (running but not implementing any task). The technique of handling idle task arc is different from that of general, as will be discussed later.

3. Reliability Model of TT&C and Communication Task Based on Markov Process

The main assumptions for the model in this paper are made as follows:

- (1) The TT&C and communication resources have two states: good or failure, their failure and repair times are independently exponentially distributed.
- (2) Repair is perfect, and always restores the resource to as-good-as-new.
- (3) The transition time between any two successive task arcs is instantaneous.
- (4) If a TTCCT fails in one task arc, the entire task is failed.

3.1 Reliability Model of TTCCT with Idle Task Arc

Assume there is a TTCCT $task_1$, its flight phase and flight cycle are fp^1 and $fc^{2,key}$ respectively. Then, the reliability model of $task_1$ can be represented as follows:

$$task_1 = \{type, fp^1, fc^{2,key}, t_1, t_4, Resources\} \quad (1)$$

$$Resources(task_1) = \{res_0, res_1, res_2, res_3, res_4\} \quad (2)$$

$$res_0 = \{r_1, t_1, t_4\}, res_1 = \{r_2, t_1, t_4\}, res_2 = \{r_3, t_1, t_2\}, res_3 = \{r_4, t_3, t_4\}, res_4 = \{r_5, t_3, t_4\} \quad (3)$$

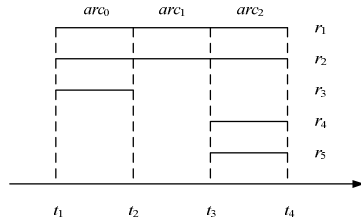


Figure 1: Time sequence of resources in $task_1$

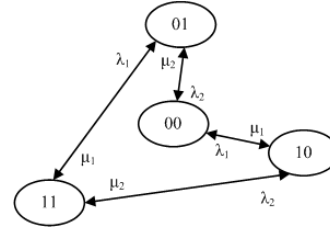


Figure 3: Markov model of arc_1 in $task_1$

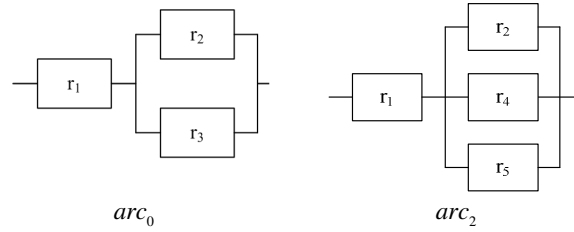


Figure 2: Reliability block diagram of non-idle arc in $task_1$

Assume that the resources r_1, r_2 are facilities of TT&C center. Time sequence of resources in $task_1$ is shown in Figure 1. $task_1$ can be divided into three task arcs, that is $Arcs(task_1) = \{arc_0, arc_1, arc_2\}$, where arc_1 is an idle task arc. Figure 2 shows the RBD of each non-idle arc. The RBD represents the system logic structure. From Figure 2 we can see that the structural configuration changes with time.

Markov model for ordinary task arc has been considered in several literature [6]. For ordinary task, the failure states of system are regarded as absorbing states as the task will fail once the system entering into these states. However, in the idle task arc interval, the system is not required to accomplish any task, so there is no absorbing state. At the same time, the component failure and repair activities are performed normally as components are still running during the idle time interval. In Figure 1, arc_1 is idle task arc, assume the failure rate and repair rate of r_i are λ_i and μ_i , the Markov model of arc_1 is given in Figure 3, where, the states are denoted as a binary string $c_1c_2...c_n$, c_k is the state of resource r_k , it take value 1 if r_k is good and 0 otherwise.

3.2 Reliability Model of TTCCTs in Multi-Flight Cycles

It is necessary to combine the TTCCTs, of the same type, in multi-flight cycles, to a new complicated one to calculate its reliability. The rules of combining TTCCTs of same

type are as follows: (i) the type and flight phase keep consistent with original TTCCTs; (ii) the flight cycle contains all cycles of original ones; (iii) the start time and the end time are the earliest start time and the latest end time of original ones respectively; (iv) the resources set is the union of original TTCCTs' resources set.

For example, for two TTCCTs $task_2$ and $task_3$,

$$task_2 = \{typeA, fp^1, fc^{4,key}, t_1, t_3, Resources\} \quad (4)$$

$$Resources(task_2) = \{(r_1, t_1, t_3), (r_2, t_1, t_2), (r_3, t_2, t_3)\} \quad (5)$$

$$task_3 = \{typeA, fp^1, fc^{5,key}, t_4, t_6, Resources\} \quad (6)$$

$$Resources(task_3) = \{(r_1, t_4, t_6), (r_2, t_4, t_5), (r_3, t_5, t_6)\} \quad (7)$$

Let $t_1 < t_2 < t_3 < t_4 < t_5 < t_6$, the combined new task can be represented as:

$$task_{new} = \{typeA, fp^1, fc^{(4,key),(5,key)}, t_1, t_6, Resources\} \quad (8)$$

$$Resources(task_{new}) = \{(r_1, t_1, t_3), (r_2, t_1, t_2), (r_3, t_2, t_3), (r_1, t_4, t_6), (r_2, t_4, t_5), (r_3, t_5, t_6)\} \quad (9)$$

As shown above, the resources set of new task has elements with same resources name. For example, names of resources in (r_1, t_1, t_3) and (r_1, t_4, t_6) are same. However, they are considered as different in the model, for their start time and end time are distinct.

According to the TTCCT reliability model, the task arcs for $task_i$ are $Arcs(task_i) = \{arc_0, \dots, arc_m, \dots\}$. Our approach adopted is to construct a continuous-time discrete-state Markov model for every arc_m , then solve each single Markov model by getting its initial conditions in such a way: the initial states probability for arc_{m+1} is set equal to the final states probability of arc_m . Then, the reliability of last task arc is the reliability of $task_i$.

4. Examples

In the following examples, assume that the mean time between failures (MTBF) and the mean time to repair (MTTR) of each resource are 30min and 20min respectively. We use the numerical method introduced in [12] to solve the Markov models of each task arc.

4.1 Idle Task Arc

For $task_1$, the RBD of each task arc is shown in Figure 2. Three task arcs are arc_0 , arc_1 and arc_2 . Assume the duration of each task arc is 10min, 30min and 15min respectively. arc_1 is idle task arc. If we delete arc_1 from $task_1$, we get a new TTCCT $task_4$. $task_1$ and $task_4$ can be described as follows:

$$task_1 = \{\text{orbit measurement}, fp^1, fc^{3,key}, 0, 55, Resources\} \quad (10)$$

$$Resources(task_1) = \{(r_1, 0, 10), (r_2, 0, 10), (r_1, 40, 55), (r_2, 40, 55), (r_3, 0, 10), (r_4, 40, 55), (r_5, 40, 55)\} \quad (11)$$

$$Arc(task_1) = \{arc_0, arc_1, arc_2\} \quad (12)$$

$$task_4 = \{\text{orbit measurement}, fp^1, fc^{2,key}, 0, 25, Resources\} \quad (13)$$

$$Resources(task_4) = \{(r_1, 0, 25), (r_2, 0, 25), (r_3, 0, 10), (r_4, 10, 25), (r_5, 10, 25)\} \quad (14)$$

$$Arc(task_4) = \{arc_0, arc_1\} \quad (15)$$

The resulting reliability of each task is shown in Table 1. We can see from Table 1 that the reliability of $task_1$ is lower than that of $task_4$, because arc_1 of $task_1$ is idle task arc, and there are still running resources during the idle task arc.

Table 1: Reliability of Task arcs of $task_1$, $task_4$ and $task_5$

$task_1$	reliability	$task_4$	reliability	$task_5$	reliability
arc_0	0.66619566	arc_0	0.66619566	arc_0	0.66619566
arc_2	0.24018238	arc_1	0.38395935	arc_1	0.38395935
				arc_3	0.11707743
				arc_5	0.03669537

4.2 TTCCTs in Multi-Flight Cycle

For $task_1$ and $task_4$, they belong to two consecutive flight cycles. Let the amount of time between $task_1$ and $task_4$ be 50min. If we focus on the task reliability of considering $task_1$ and $task_4$ as a single TTCCT, then we can combine them and get a new task $task_5$. According to the combination rule given in section 3.2, $task_5$ can be described as follows:

$$task_5 = \{\text{orbit measurement}, fp^1, fc^{(2, key), (3, key)}, 0, 130, Resources\} \quad (16)$$

$$Resources(task_5) = \{(r_1, 0, 25), (r_2, 0, 25), (r_3, 0, 10), (r_4, 10, 25), (r_5, 10, 25), (r_1, 75, 85), (r_2, 75, 85), (r_1, 115, 130), (r_2, 115, 130), (r_3, 75, 85), (r_4, 115, 130), (r_5, 115, 130)\} \quad (17)$$

$$Arc(task_5) = \{arc_0, arc_1, arc_2, arc_3, arc_4, arc_5\} \quad (18)$$

arc_2 of $task_5$ is an idle task arc because it is the interval process between $fc^{2, key}$ and $fc^{3, key}$, and during this time there is no required task to be performed. arc_0 and arc_1 of $task_5$ are actually the two task arcs in $task_4$, and arc_3 , arc_4 and arc_5 of $task_5$ are corresponding to the three task arcs of $task_1$. The resulting reliability of $task_5$ are shown in Table I.

Comparing the reliability of task arcs of $task_1$, $task_4$ and $task_5$ in Table I, We can see that after the idle task arc arc_2 , the resulting reliability of $task_5$ is declined evidently. As the interval between two tasks in consecutive flight cycles is generally a long time, restarting the resources during this time to restore them to as-good-as-new is important for improving the task reliability.

5. Conclusions

In this paper, we study the reliability modeling of the TT&C and communication task. According to the example results, although an idle task arc contains no absorbing state, it affects the task reliability especially when its lasting time is long enough. If the resources can be restarted during the idle task arc, the task reliability will be improved. But in this situation, the preparing time after resource opening and the resource close time must be considered in models. Therefore, future research work is needed to deal with resource restarting situation.

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