Multi-Criteria Decision Model for Imperfect Maintenance using Multi-Attribute Utility Theory

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\begin{abstract}
Many research works have been conducted in the preventive maintenance area since maintenance strategies have become more and more significant in industry and supply chain services. However, previous studies are mainly based on age maintenance policies and other multi-criteria approaches instead of Multi-Attribute Utility Theory (MAUT). There are some studies proposed by using MAUT, but they assume that the maintenances are perfect. This paper presents an imperfect maintenance model of a one-unit system to obtain an optimal inspection interval based on MAUT. The proposed model is designed to identify the systems’ status by making a trade-off between cost attribute and reliability attribute. By taking decision makers’ preferences into account, with the assumption of imperfect maintenance, the model receives a result of an optimal inspection interval. This model is applicable for equipment or systems that suffer graded failures and can be repaired at any time during the operation. A numerical application is given to illustrate that with consideration of decision-makers’ priorities, the model can provide new solutions for imperfect maintenance interval optimization, which is a suggestion for future research.

\textit{Keywords:} imperfect maintenance; optimal intervals; reliability; cost; multi-attribute utility theory

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\end{abstract}

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1. Introduction

Systems used in commodity production and delivery services are subject to deterioration with usage and age. This causes higher production costs and lower product output with deterioration, erosion, and so on [1]. Preventive maintenance is an effective way to prevent failures of systems as failure rates increase with service time. Moreover, it can prevent productivity from decreasing. Many models based on preventive maintenance policies assumed that systems become as good as new after preventive maintenance. In Yang’s paper, a perfect preventive maintenance model is proposed based on age-based replacement policy to obtain an optimization. In his research, a three-stage failure process is considered dividing the lifetime of the system into four states. When a defective stage is identified or at failure, the system is renewed. Moreover, the system is replaced once a certain age $T$ is reached [2]. In Wang’s paper, a joint optimization for spare parts inventory and preventive maintenance inspection interval is presented. The model develops a delay-time concept in the situation of perfect maintenance [3]. Wang also proposes several perfect maintenance models using the delay-time concept, such as a block-based inspection model that uses a recursive algorithm for determining a limiting distribution in order to optimize the inspection interval [4].

However, perfect maintenance is not true in practice. In fact, systems may be as bad as before or become a little worse than new after preventive maintenance due to wrong adjustments, bad parts, and damage done during preventive maintenance [5]. This kind of maintenance is known as imperfect maintenance. A precise definition of imperfect preventive maintenance is as follows: any maintenance action that makes a system “younger” and results in an improved system operating condition [1]. Recently, issues of imperfect maintenance have drawn a large amount of attention, and much research has been conducted based on them. Some researchers focus on imperfect preventive maintenance problems under the situation of delay-time concept. Yang modifies Wang’s three-stage delay-time failure process and puts imperfect
Multi-criteria decision making, as one of the most widely applied methods in the decision-making area, is spread over many fields nowadays. A few works focus on preventive maintenance issues based on methods of multi-criteria decision making. For example, with regard to decision-making, Shan proposes a multi-objective maintenance model and uses the Dempster-Shafer evidence theory. The change of distribution system reliability after maintenance, load loss, and maintenance cost are three indexes in the model [11]. Still, there exist some studies researching decision-making model based on imperfect maintenance. In one study, Preference Ranking Organization Methods for Enrichment Evaluations (PROMETHEE), one method of decision-making, is used to select the optimal maintenance interval by improving the equipment reliability and minimizing total maintenance costs [12]. According to Ana’s paper, unavailability and cost are two conflicting decision criteria for determining imperfect maintenance optimization [13].

Multi-attribute utility theory (MAUT) is a branch of multi-criteria decision making that concerns modeling utility functions with multiple attribute outcomes and obtaining the best choice among different options [14]. However, there have not been many articles taking decision makers’ preferences into account based on MAUT. Even though some papers explore maintenance interval optimization problems based on MAUT approaches, they do not consider imperfect preventive maintenance situations [15]. According to Garmabaki and Ahmadi, maintenance decision making may occur in various complex systems subject to technology, maintainability, reliability and availability requirements, etc. In their study, three attributes, cost, reliability, and availability, are considered in the model. They assume that performing preventive maintenance or repair will take some time. However, they only consider the situation of perfect preventive maintenance [16]. In Adiel’s paper, some constraints are reduced but perfect maintenance is still assumed.

In this paper, we propose a multi-criteria decision model for imperfect maintenance based on MAUT. Dealing with two uncertain variables, decision-makers’ preferences are taken into account and the inspection interval is optimized. Different preference uncertain attributes, cost and reliability, are considered in our utility functions. This paper aims to determine an optimal imperfect preventive maintenance interval by improving reliability and reducing total cost during the operation of the system. We then illustrate an application to analyze a practical problem and make a discussion based on it.

The remaining parts of this paper are organized as follows. Section 2 provides the problem description and assumptions. In this section, the imperfect maintenance policy is presented and the cost model is proposed. Moreover, the decision model is formulated based on multi-attribute utility theory. Then, Section 3 gives a numerical example to illustrate the applicability of the model, and the obtained results are analyzed. Finally, Section 4 provides a conclusion that the multi-criteria decision model is effective in solving problems for imperfect maintenance.

2. Model Assumptions and Description

2.1. Assumptions

As we know, failures can be classified into catastrophic failures, which mean systems fail all of sudden, and degraded failures, which refer to systems failing due to performance deterioration. The proposed model is based on degraded failures in this paper. Some systems can only accept maintenance actions before given tasks, that is, defects may be tested while the systems are in storage. This includes rocket engines, missiles, and so on [17]. Here, we only study the situation that maintenance actions can be taken at any time. A one-unit system is used in our case. It can be representative of equipment, a component, or multi-unit systems in which failures can be detected by inspections. In many papers, researchers define action space as a continuous set and generate a continuous consequence space in order to obtain smooth curves. However, this is impossible in real life. Preventive maintenance is always carried out in a period, such as one day, two weeks, three
months, etc. Taking the reality into account, the action space and consequence space in our paper are defined as discrete sets of alternatives.

The periodical inspections to detect defects or failures may not be perfect. Preventive maintenance is also imperfect. In this paper, we assume the system becomes as good as new only after perfect preventive maintenance or after repairing, which means that maintenance is carried out just after failure occurs. Utilities are considered additive independent. We also assume the unit has the same failure rate before or after preventive maintenance. We define a gamma distribution as our failure distribution. According to assumptions of our imperfect maintenance method, all maintenance actions take negligible time.

The notations used in the following text are listed below:

- $T$: Time at which the operating unit is repaired at failure or is preventively maintained
- $P$: $0 \leq p < 1$, is the probability that the unit has the same failure rate after preventive maintenance as it had before maintenance
- $q$: $q = 1 - p$, which refers to the unit becoming as good as new after preventive maintenance
- $f(t)$: Probability density function of failure
- $F(t)$: Cumulative distribution function of $f(t)$ with mean value $\mu$
- $R(t)$: Reliability function, namely $R(t) = 1 - F(t)$
- $c_i$: Cost of each corrective maintenance
- $c_p$: Cost of each preventive maintenance
- $E(c)$: Expected cost rate
- $w_c$: Weight parameters for cost attribute
- $w_r$: Weight parameters for reliability attribute
- $r$: Random variable of cost
- $c$: Random variable of cost
- $k$: Constants that keep numerical values $U(x)$ ranging from 0 to 1. $i=1, 2, 3, \ldots$
- $U(C)$: Single utility functions for cost
- $U(R)$: Single utility functions for cost
- $U(x_1, x_2, \ldots, x_n)$: Namely $U(C, R)$, multi-attribute utility function

2.2. Multi-Attribute Utility Theory (MAUT)

Generally, MAUT is defined as

$$U(x_1, x_2, \ldots, x_n) = f[u_1(x_1), u_2(x_2), \ldots, u_n(x_n)] = \sum_{i=1}^{n} w_i u_i(x_i)$$  

Where $\sum_{i=1}^{n} w_i = 1$.

In this paper, we consider two attributes: cost attribute and reliability attribute. According to Jansen’s paper [18], we assume utility functions are additive independent. Then, the function based on MAUT is given by

$$\text{Max}: U(C, R) = w_c U(C) + w_r U(R)$$  

Where $w_c + w_r = 1$.

By maximizing this multi-attribute utility function, the optimal inspection $T^*$ will be obtained by maximizing the function above.
2.3. The Cost Model & Cost Attribute

First, we define a cycle as a period of time that begins from a perfect situation of the unit and ends in a failure. Then, the average cost in a cycle based on optimal preventive replacement is given by

\[ C(T, p) = \frac{E(c)}{E(T)} \]  

(3)

According to the assumptions above, the probability of repairing at failure is

\[ \sum_{j=1}^{\infty} p^{j-1} \int_{(j-1)T}^{jT} dF(t) = 1 - q \sum_{j=1}^{\infty} p^{j-1} R(jT) \]  

(4)

Where \( j = 1, 2, 3, \ldots \)

The expected preventive maintenance number in one cycle is

\[ \sum_{j=1}^{\infty} (j-1) p^{j-1} \int_{(j-1)T}^{jT} dF(t) + q \sum_{j=1}^{\infty} j p^{j-1} R(jT) = \sum_{j=1}^{\infty} p^{j-1} R(jT) \]  

(5)

Moreover, we can easily obtain the function of meantime of a single cycle, that is

\[ \sum_{j=1}^{\infty} p^{j-1} \int_{(j-1)T}^{jT} dF(t) + q \sum_{j=1}^{\infty} j p^{j-1} R(jT) = \sum_{j=1}^{\infty} p^{j-1} \int_{(j-1)T}^{jT} R(jT) dt \]  

(6)

Above all, \( C(T, p) \) is given by

\[ C(T, p) = \frac{c_j [1 - q \sum_{j=1}^{\infty} p^{j-1} R(jT)] + c_p \sum_{j=1}^{\infty} p^{j-1} R(jT)}{\sum_{j=1}^{\infty} p^{j-1} \int_{(j-1)T}^{jT} R(t) dt} \]  

(7)

From Equation (5), a minimum \( C_{\text{min}} \) can be calculated. According to Garmabaki’s paper, the cost attribute function is

\[ U_{\text{Cost}} = \frac{C_{\text{min}}}{C(T, p)} \]  

(8)

2.4. Reliability Attribute

A reliability function is obtained since we know the probability density function of failure \( f(t) \).

\[ R(t) = \Pr(T \geq t) = 1 - F(t) \]  

(9)

From the properties of reliability distribution, we know that the larger the value of \( R(t) \), the better the obtained results. The reliability attribute is given by

\[ U_{\text{R}} = \frac{R(T)}{R_{\text{Max}}} \]  

(10)

2.5. Decision Modeling of Single Utility Function

Decision makers’ preferences are taken into account to analyze our model according to decision makers’ behaviors towards
risk. From Almeida’s paper, utility functions can be classified into three forms: linear form and two exponential forms.

The linear utility function is as follows:

\[ U(x) = k_1 x + k_2 \]  

(11)

The exponential utility functions are as follows:

\[ U(x) = k_1 \exp\left(-\frac{k_2}{x}\right) \]  

(12)

and

\[ U(x) = k_2 \exp(-k_3x) \]  

(13)

It can be seen easily that when the decision maker is risk neutral, the linear utility function is suitable for the model; when the decision maker is risk averse, the exponential utility function is recommended. For the cost attribute in this paper, the linear utility function is applied, and the logistic utility function is applied for the reliability attribute.

That is, for the cost attribute:

\[ U(c) = k_1 c + k_2 \]  

(14)

For the reliability attribute:

\[ U(r) = k_1 \exp\left(-\frac{k_2}{r}\right) \]  

(15)

3. Numerical Examples

In order to demonstrate the practicability of the proposed model, a numerical application is illustrated to show details about how the model works. We assume the failure distribution of one unit system obeys the gamma distribution with parameters \( \alpha \) and \( \lambda \). Therefore, it can be indicated by

\[
p(T; \alpha, \lambda) = \begin{cases} \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-\lambda x}, & T \geq 0 \\ 0, & T < 0 \end{cases}
\]  

(16)

Then, we establish the action space. According to the assumptions we made above, the action space is defined as five days to satisfy a discrete condition. That is,

\[ t_i = 5, t_{i+1} = 10, \ldots, t_{5i} = 5i, i = 1, 2, 3, \ldots \]  

(17)

We also define \( H(t; p) \) as

\[
H(t; p) = \frac{\sum_{j=t}^{\infty} p^{j-1} f(jt)}{\sum_{j=t}^{\infty} p^{j-1} j(1-F(jt))}
\]  

(18)

In order to generate a unique finite optimal inspection interval \( T^* \) according to unique solution conditions of imperfect preventive maintenance [3], values of the parameters should satisfy:

\[ c_j q > c_p \]  

(19)
and

\[ H(\alpha; p) > c_j q / [\mu(c_j q - c_p)] \quad (20) \]

Considering the conditions above and data of real examples in other papers [13,15], we set simulated data to parameters as below. They are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>2</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1</td>
</tr>
<tr>
<td>( c_p )</td>
<td>600</td>
</tr>
<tr>
<td>( c_j )</td>
<td>3000</td>
</tr>
<tr>
<td>( q )</td>
<td>0.95</td>
</tr>
</tbody>
</table>

The failure cumulative distribution function can be obtained by given \( \alpha \) and \( \lambda \), as drawn in Figure 1.

It can be easily verified that the given data above satisfies Equation (19) and Equation (20) by taking the specific values into these two equations.

After verification, we put the simulated data into Equation (7) to get the values of costs versus time. Thus, a modified cost function \( C(T, p) \) can be obtained as below.

\[
C(T, 0.05) = \frac{3000 - 2250 \sum_{j=1}^{\infty} (0.95)^{j-1}(1 + jT)e^{-jT}}{\sum_{j=1}^{\infty} (0.95)^{j-1}\int_{j-1}^{jT} (1 + jte^{-\mu t}) dt}
\]

(21)

Thus, a graph through the curve of cost versus time is easily obtained as Figure 2 by running MATLAB.

It can be seen that from the first four days, the cost rate has a tendency to decline rapidly. Then, it declines gradually and almost approximates a specific number as the value of time increases. The values are expressed in Table 2.

From Table 2, it can be seen that the values of the cost reach the minimum when \( i \) is greater than or equal to 11. This is because we set the reservation arrives decimally hind four in the calculation of \( C(T, 0.05) \).

From the description of the limiting conditions above, we know that the model satisfies one unique solution condition in Equations (19) and (20). Therefore, there must be one unique result in our case study. In order to see clearly, we extend the time of interval \( T \) and plot the figure again. At the same time, we set the reservation arrives decimally hind four and
increase the values by multiplying by 1000. Then, the results arrived at in Figure 3 show that an optimal inspection interval $T^*$ is obtained as expected.

<table>
<thead>
<tr>
<th>i</th>
<th>value</th>
<th>i</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0e+03</td>
<td>11</td>
<td>0.9828</td>
</tr>
<tr>
<td>1</td>
<td>2.849</td>
<td>12</td>
<td>0.9828</td>
</tr>
<tr>
<td>2</td>
<td>1.1696</td>
<td>13</td>
<td>0.9828</td>
</tr>
<tr>
<td>3</td>
<td>1.0662</td>
<td>14</td>
<td>0.9828</td>
</tr>
<tr>
<td>4</td>
<td>1.0188</td>
<td>15</td>
<td>0.9828</td>
</tr>
<tr>
<td>5</td>
<td>0.9981</td>
<td>16</td>
<td>0.9828</td>
</tr>
<tr>
<td>6</td>
<td>0.9892</td>
<td>17</td>
<td>0.9828</td>
</tr>
<tr>
<td>7</td>
<td>0.9854</td>
<td>18</td>
<td>0.9828</td>
</tr>
<tr>
<td>8</td>
<td>0.9838</td>
<td>19</td>
<td>0.9828</td>
</tr>
<tr>
<td>9</td>
<td>0.9832</td>
<td>20</td>
<td>0.9828</td>
</tr>
<tr>
<td>10</td>
<td>0.9829</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Though it seems that cost rate approaches a specific value as time goes by in Figure 3, this curve still has its lowest point by calculation. As illustrated at the top of Figure 3, the optimal cost rate is approximately equal to 983 at the optimal inspection interval $i^* = 39$. Moreover, we assume the action space is discrete, namely, the assume inspection is carried out in five days. Still, the result approaches a smooth curve in the figure and keeps the integrity of its tendency.

As we calculated above, the highest cost rate is approximately equal to 2849 when $i$ equals 0 and the lowest is 982.72 when $i$ equals 39. Thus, from Equation (14), we conduct an elicitation procedure of cost rate. Then, we can easily obtain $k_1 = -5.36 \times 10^4$ and $k_2 = 1.53$.

According to Equation (9), we can observe the expression of $R(t)$ based on the expression of the failure cumulative distribution. Figure 4 displays the tendency of $R(t)$.

Then, according to Equation (15), we conduct an elicitation procedure and obtain the results $k_3 = 1.01$ and $k_4 = 0.01$.

From the above, we get

$$
\begin{align*}
U(c) &= -5.36 \times 10^4 c + 1.53 \\
U(r) &= 1.01 \exp(-0.01/r)
\end{align*}
$$

Then, the Equation (22) can be easily drawn as Figure 5 and Figure 6.
As illustrated in Figure 5, the higher the reliability of the system or equipment, the higher the utility and the more satisfaction the decision makers will get. If the reliability equals 1, the utility reaches its maximum value of 1. If the reliability of the system or the equipment is equal to 0, the corresponding utility is 0. From the other side, Figure 6 shows that the higher the system or equipment cost, the less utility it will obtain.

Considering a case where the reliability is 1.5 times more important than the cost rate, then combine $U(C)$ and $U(R)$ with $W_c=0.6$ and $W_R=0.4$. We can obtain Equation (23) as below.

$$U(C, R) = -3.22 \times 10^{-4} c + 0.404e^{-0.01/r} + 0.92$$

Then, the graph of $U(C, R)$ is illustrated as below.

As displayed in Figure 7, a unique optimal inspection interval exists and received at $i \in [2, 5]$, that is, $T \in [10, 25]$. By calculation, the optimal utility equals 0.9593. It arrives at the maximum value when $i$ approximates to 4, which means that the optimal maintenance plan of the system or equipment is to conduct a maintenance action about every 20 days.

4. Sensitivity Analyses of Parameters

According to Table 1, the calculations above are based on settings where $\alpha=2$, $\lambda=1$, $C_p=600$, $C_f=3000$, and $q=0.95$. Furthermore, we set weight parameters $w_c=0.6$ and $w_R=0.4$. If we change the values of any parameters, the results may be changed. Here, we give some examples to illustrate the sensitivity of these parameters.

For example, if we reset weight parameters $w_c=0.8$ and $w_R=0.2$, then the optimal inspection time is $T \in [15, 25]$ and the optimal utility equals 0.9682.

The results of changed weights are graphed by Figure 8. It is apparent that the tendency is still the same as that seen in Figure 7. However, the values of utility tend to different balance points.
From Equation (1), it is implied that the utility functions are additive independent. If we weaken the condition, the utility function can be built as Equation (24) or other equation forms.

\[
\text{Max}: U(C, R) = w_c U(C) + w_R U(R) + w_{CR} U(C)U(R)
\]

(24)

With other conditions unchanged, if we set the weight parameters as \( w_c = 0.3 \), \( w_R = 0.2 \), and \( w_{CR} = 0.5 \), we obtain the optimal value of utility as 0.9448 by using Equation (22).

From the other aspect, risk preferences of decision makers may be different from assumptions in our case study. For example, a decision maker is risk averse to cost attributes, and then the utility function of cost attribute should be expressed as an exponential form.

It can be assumed as the function below.

\[
U'(c) = k_c \exp\left(-\frac{k_c}{c}\right)
\]

(25)

According to the analysis in Section 3, following the elicitation procedure, a modified cost utility function is obtained. If we keep the utility function of reliability the same as in the previous assumption, they can be written as

\[
\begin{align*}
U'(c) &= 2.3 \times 10^{-20} \exp(4.44 \times 10^3 / c) \\
U'(r) &= 1.01 \exp(-0.01 / r)
\end{align*}
\]

(26)

According to the equations set, we set weight parameters \( w_c = 0.6 \) and \( w_R = 0.4 \), and then \( U'(c, r) \) can be obtained, that is

\[
U'(c, r) = 1.38 \times 10^{-20} e^{4.44 \times 10^3 c} + 0.404 e^{-0.01/r}
\]

(27)

The graph is shown below in Figure 9. The maximum value of utility is 0.6743 from calculation, which is lower than the situation of a linear utility function. This is because it is easier for risk neutral to be satisfied than risk averter.

Above all, the comparison of calculated functions is shown in Figure 10.

![Figure 9. Modified utility function](image)

![Figure 10. Compared values of utility functions](image)

As can be seen from the graph, at the beginning, all utility values of cost risk neutral sharply increase as the interval time gets larger. Then, they respectively reach their maximum values. Though different maximum values are obtained, they follow the same tend. From the perspective of cost risk aversion, though the tendency at the beginning is different from risk neutral ones, there is still only one maximum point.
In general, the model and structure are based on real situations of complex systems or equipment. The model is applicable to systems that suffer graded failures and can take maintenance actions at any time during the operation. Therefore, it can be applied to factories that focus on manufacturing production, companies that provide service production, and so on.

5. Conclusions

In this paper, we build a multi-attribute utility function and find a unique optimal inspection interval of imperfect preventive maintenance. The cost attribute and the reliability attribute are two main indexes in our model. We estimate the cost rate using Equation (7). As discussed above, it is assumed that the failure of the system or equipment follows the gamma distribution. In order to conduct a unique optimal interval, parameters have to satisfy the limit conditions of Equation (19) and Equation (20). Then, we use a linear function and an exponential function respectively representing cost and reliability utility functions. We assume that the utility functions mentioned are additive independent, which means that they can be estimated by using Equation (1).

From the numerical examples in Section 3, a unique optimal inspection interval is obtained. Moreover, values of some parameters are changed to analyze the sensitivity of the proposed model. This numerical application has illustrated that the use of MAUT in imperfect preventive maintenance is applicable. In conclusion, this model determines a new solution to optimize an optimal interval of imperfect preventive maintenance with the consideration of decision-makers’ preferences.

Further studies may be carried out on the analysis of a modified model that contains more considered attributes. More complex but accurate methods may be used to express actions of imperfect preventive maintenance. It may also be conducted from the aspect of whole supply chain management.

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