Residual Life Prediction of Long-Term Storage Products Considering Regular Inspection and Preventive Maintenance

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Abstract

Predicting the residual life of long-term storage products with regular inspection and preventive maintenance is of great significance nowadays. In this paper, a model of storage process that takes multi-stage degradation and preventive maintenance into consideration is established. Considering the amount of degradation of the product to follow the Wiener process, we put forward a method to predict the residual life of long-storage products based on the degradation model in a multi-stage storage process. Through a simulation method, five experiments are performed to calculate and compare the residual life in different situations. Finally, we find that the dramatic changes of environmental conditions during the inspection period influence the residual storage life observably. By simulation, this model is effective by making full use of data collected during storage time, including degradation amount and maintenance information.

Keywords: multi-stage degradation; preventive maintenance; residual storage life; Wiener process

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1. Introduction

Residual life prediction of long-term storage products is an important problem currently, especially considering regular testing and preventive maintenance, and it is significant for product health management and equipment production. During the storage time, which consists of the storage period and the inspection period, performance degradation exists in some of the products. The storage period is long, and degradation increases slowly. The inspection period is short, and degradation increases obviously.

For the residual life prediction of specific service equipment, Gao [1] assumed that there is a linear dependence between the exponent ratio and the loading ratio to predict fatigue residual life of materials based on the nonlinear fatigue damage accumulation model. For electromagnetic relays, Zhao [2] proposed a particle filtering-based method for predicting their remaining storage life, which was proven to be effective. Zhang [3] proposed a new residual life prediction method for complex systems based on the Wiener process and evidential reasoning. However, the method is difficult. Based on a similarity-based approach, Blaise [4] used degradation observations depending on acquisition time and reference dataset information built on the knowledge of endurance degradation data to perform prediction. Son [5] conducted a comparative analysis of various residual life prediction methods based on the random coefficient regression model. It is difficult for these methods to reflect the concept of first-time, and this may affect the forecast results.

Regarding maintenance in the degradation process of products, many studies have been carried out. Du [6] proved that timely external maintenance and a sufficient supply of electrolytes can greatly extend the lifespan of storage batteries. Cherkaoui et al. [7] dealt with a quantitative approach to jointly assess the economic performance and robustness of some representatives of time-based and condition-based maintenance. In the maintenance model of Komijani [8], an investigation on the concurrent effects of random shocks during the useful life of the equipment was also studied. Zhang [9] considered the effect of maintenance on the parameters of the degradation process, analysed the residual life of the product under imperfect maintenance, and made maintenance decisions. Yang [10] considered a class of systems with two failure modes,
both of which were taken into consideration in the impact of different states of the system. Nourelfath [11] allowed for a joint selection of the optimal values of production plan and the maintenance policy, while taking into account quality-related costs. The research of Lee [12] was based on the assumption that the failure process between two preventive maintenances follows a generalized version of the nonhomogeneous Poisson process. The limitation of these studies is that almost all of them only consider maintenance in a single process.

Considering the multi-stage process, Park [13] believed that it is essential for the multi-stage process monitoring to be able to give a signal at each single stage in order to avoid the delay in detecting assignable causes in the process. Zheng [14] established a staged degradation process model that can describe the effects of incomplete maintenance. Sheng [15] proposed an autoregressive moving average model-filtered hidden Markov model to fit the multi-phase degradation data with an unknown number of jump points. In the current research, most of the studies were conducted to optimize the repair strategy at the same time as the assessment. However, we concentrate on a multi-stage degradation process based on the fixed maintenance strategy.

Overall, in this paper, we analyse the multi-stage degradation process model of long storage products that are regularly tested and repaired, taking full account of the impact of different environmental stresses on the degradation rate of a single product. The simulation method is used to solve the model and determine the residual storage life of products with different degradation parameters. Focusing on the degradation model in multi-stage storage process, we propose a method to predict the residual life of long storage products when considering the amount of degradation of the products to follow the Wiener process. Therefore, this paper provides a way of thinking for the residual life study of long-term storage products considering regular inspection and preventive maintenance.

The rest of the paper is structured as follows. In section 2, a symbol description is presented. Then, we establish the storage model of a single product considering regular inspection and maintenance in section 3. In section 4, we study the prediction of residual life to storage of a single product with regular inspection and preventive maintenance. In section 5, a simulation method is applied to solve the residual life. Finally, the conclusions are given in section 6.

2. Nomenclature

\( \sigma \) the diffusion coefficient of the Wiener process
\( \nu(t) \) the state of the component
\( \mu(\nu(t)) \) the rate of degradation of the part
\( \lambda(t) \) the stochastic process of degeneracy
\( B(t) \) the standard Brownian motion
\( T_i \) the storage time at the moment
\( T \) the length of storage period
\( d \) the length of inspection period
\( t_m \) the time for maintenance
\( N_m \) the number of repairs from the beginning of storage to product being failed
\( k_i \) the number of repairs at time \( T_i \)
\( k \) the maximum number of preventive maintenance
\( M_1 \) the failure threshold
\( M_2 \) the maintenance threshold
\( \mu_1 \) the degradation rate during storage period
\( \mu_2 \) the degradation rate during detection period
\( \mu_m \) the maintenance effect
\( \sigma_m \) the diffusion coefficient of the Wiener process in maintenance period
\( L \) the point estimation of residual storage life


3.1. Problem Description

In the long-term storage of large-scale equipment systems, performance degradation during storage exists in some of the products, resulting in a direct impact on the availability of products.
The multi-state degradation process includes the storage period and the inspection period. The storage period is a long state when degradation increases slowly. The inspection period is a short state when degradation increases obviously.

At the same time, the structure of this kind of equipment is relatively simple in design and easy to disassemble and install. Therefore, maintaining high availability of products during storage may require several repairs or replacements. The multi-state storage process of product is shown in Figure 1.

![Degradation process in multi-state storage period](Image)

To sample the problem, we make some basic assumptions as follows:

- The maintenance operation takes up negligible time during the entire storage process. In this paper, the main body of the storage process is a natural storage stage, and the process of inspection in it has been relatively short. As an operation in the inspection process, maintenance can be regarded as being completed immediately.
- Maintenance does not affect the parameters of the degradation process. The effect of maintenance on the degradation process is only reflected in the performance degradation values.
- Maintenance can occur at any one moment in the inspection process. Products after the repair do not change the storage state, and their storage state at any time is only related to the pre-determined storage strategy.

### 3.2. Degradation Model

Due to the good nature of the Wiener process, we use a single-unit linear Wiener process with drift to model the multi-state storage process. Referring to the model description of Si et al. [16], \( v(t) \) denotes the state of the component at time \( t \), where \( v(t) = 1 \) indicates that the product is in a natural storage state and \( v(t) = 2 \) indicates that the product is in an inspection state. Then, \( \mu(v(t)) \) denotes the rate of degradation of the part at time \( t \). Let the stochastic process \( \{X(t), t \geq 0\} \) represent the process of degeneracy, in which \( X(t) \) represents the amount of degradation at time \( t \). Then, we assume

\[
X(t) = x(0) + \int_0^t \mu(v(u)) du + \sigma B(t)
\]

(1)

Where \( \sigma > 0 \) denotes the diffusion coefficient of the Wiener process and \( \{B(t), t \geq 0\} \) denotes the standard Brownian motion.

We assume that the inspection start time and the duration in this problem are both fixed values. The natural storage state and the inspection state can be regarded as linear Wiener processes. Therefore, the degradation process can be expressed as the following form:

\[
X(t) = \begin{cases} 
\mu_1 t + \sigma B(t), & (k-1)d \leq t < kT + (k-1)d \\
\mu_2 t + \sigma B(t), & kT + (k-1)d \leq t < kT + kd 
\end{cases} , \quad k = 1,2,\ldots , n
\]

(2)

Where \( T \) denotes the length of the storage period, \( d \) denotes the length of the inspection period, and \( \mu_1 \) and \( \mu_2 \) respectively denote the degradation rate during the degradation period and the maintenance period. According to the above degradation process, the lifetime of product \( T \) can be explained as the first passage time that the degradation reaches the failure threshold.
Therefore, on the condition of knowing the current performance inspection data, the distribution function of the lifetime of products can be expressed as the following conditional distribution function:

\[ P(T \leq t) = P(\sup_{t>0}(X(t) \geq \omega)|X_i) \]  

From Formula (3) and Formula (4), the residual life of products when stored to time \( T_i \) can be expressed as

\[ S_i = \inf\{s_i; X(T_i + s_i) \geq \omega\} \]  

Similarly, the residual life distribution function of products is

\[ P(S_i \leq s_i|X_i) = P(\sup_{s_i>0}(X(T_i + s_i) \geq \omega)|X_i) \]  

Under the parameter callback method, with \( t_m \) denoting the time for maintenance, the effect of this maintenance can be described as

\[ x(t_m + 1) = x(t_m) - \Delta m \]  

Where \( \Delta m \sim N(\mu_m, \sigma_m^2) \) represents the callback amount of the performance degradation parameter after maintenance.

In actual processing, the parameters in the callback amount distribution need to be determined according to the historical maintenance data and the maintenance data of the same type of product. The presentation of the maintenance method is shown in Figure 2.

![Figure 2. Maintenance in storage period](image)

### 3.3. Estimation of Parameters

Under the parameter callback method, the maintenance parameters that need to be estimated are \( \mu_m \) and \( \sigma_m^2 \). The data used in the estimation process is the change amount of the performance parameter of a single product before and after the maintenance to the time \( T_i \), and the effect of the \( k \)-th maintenance is \( d_{m_k} (k = 1, 2, \cdots, k_i) \), where

\[ d_{m_k} = X(t_{m+k} + 1) - X(t_{m+k}) \]  

Then, the Bayesian estimation method is used to estimate the distribution parameters of the maintenance effect with the use of current maintenance effect data.

Since the distribution of \( \Delta m \) is known, the likelihood function can be easily obtained. Let the probability density function of \( \Delta m \) be

\[
f(\Delta m | \mu_m, \sigma_m^2) = \frac{1}{\sqrt{2\pi} \sigma_m} \exp \left( - \frac{(\Delta m - \mu_m)^2}{2\sigma_m^2} \right) \]

Assume that the maintenance effect parameters $\mu_m$ and $\sigma_m^2$ are independent of each other. Hence, with the samples $D_{mk} = \{dm_1, dm_2, \cdots, dm_k\}$, the likelihood function is

$$L(\mu_m, \sigma_m^2) = \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{(dm_i - \mu_m)^2}{2\sigma_m^2}\right)$$  \hspace{1cm} (10)

Therefore, we can get the probability density function of $D_{mk}$.

$$f(D_{mk}|\mu_m, \sigma_m^2) = \prod_{i=1}^{k} \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{(dm_i - \mu_m)^2}{2\sigma_m^2}\right)$$  \hspace{1cm} (11)

Next, the prior distribution should be determined. Under the condition of no information, when the distribution of samples is normally distributed, the prior distributions of mean and variance are non-information distribution. Assume that the prior distributions of $\mu_m$ and $\sigma_m^2$ are both evenly distributed, where

$$\mu_m \sim U(\mu_{m1}, \mu_{m2})$$

$$\sigma_m^2 \sim U(\sigma_{m1}^2, \sigma_{m2}^2)$$

Due to the assumption of independence, the joint prior density function of $\mu_m$ and $\sigma_m^2$ is

$$\pi(\mu_m, \sigma_m^2) = \frac{1}{(\mu_{m2} - \mu_{m1})} \times \frac{1}{(\sigma_{m2}^2 - \sigma_{m1}^2)}$$  \hspace{1cm} (12)

With the prior distribution and likelihood function being obtained, the Bayesian principle can be used to obtain the joint posterior distribution density function of the parameters $\mu_m$ and $\sigma_m^2$, that is:

$$f(\mu_m, \sigma_m^2|d) = \frac{\pi(\mu_m, \sigma_m^2)f(D_{mk}|\mu_m, \sigma_m^2)}{\int_\theta \pi(\mu_m, \sigma_m^2)f(D_{mk}|\mu_m, \sigma_m^2)d\theta}$$  \hspace{1cm} (13)

Where $\theta$ represents the parameter vector $(\mu_m, \sigma_m^2)$ to be estimated.

If it is necessary to estimate a specific parameter, the joint posterior distribution density function of the parameter $(\mu_m, \sigma_m^2)$ will be integrated for the rest of the parameters to obtain the edge posterior density of the parameter that needs to be estimated. After that, the density function can be used to solve the expectation of the parameter to be estimated, and the point estimation value as a parameter is substituted into the subsequent solution. On the other hand, the density function can also be sampled and combined with the simulation algorithm for the residual storage life, which can receive more accurate prediction results.

4. Residual Storage Life Prediction

4.1. Prediction Model

In view of the existence of a preventive maintenance ceiling, the distribution of the residual storage life needs to be discussed based on the number of repairs. For a single product, assuming that the maximum number of preventive maintenance is $k$, its lifetime is given by the total probability formula as

$$P(T \leq t) = \sum_{j=0}^{k} P(T \leq t|N_m = j) \cdot P(N_m = j)$$  \hspace{1cm} (14)

Where $N_m$ denotes the number of repairs from the beginning of storage to the product being failed. As can be seen from the analysis in the introduction, failures can occur during any natural storage period, so the number of repairs can be any integer value from 0 to the upper limit.
When the product is stored at time $T_i$, it may have been repaired several times and recorded as $k_i$. Therefore, the number of repairs and the change of product degradation after each service can be regarded as the maintenance data stored until $T_i$. Similarly, the performance inspection data during storage can be regarded as the condition for analysing residual life at $T_i$. Then, the residual lifetime at $T_i$ can be expressed as

$$P(S_i \leq s_i|X_i) = \sum_{j=0}^{k-k_i} P(S_i \leq s_i|X_i, N_{m,i} = j) \times P(N_{m,i} = j)$$ (15)

Where $N_{m,i}$ denotes the number of repairs of product after $T_i$.

$$P(S_i \leq s_i|X_i) = P(\sup_{s_i+0}(X(T_i + s_i) \geq \omega_2)|X_i)$$ (16)

Hence, the single residual storage life distribution can be expressed as

$$P(S_i \leq s_i|X_i) = \sum_{j=0}^{k-k_i} P(\sup_{s_i+0}(X(T_i + s_i) \geq \omega_2)|X_i, N_{m,i} = j) \times P$$ (17)

To solve Formula (17), the number of repairs to product failure after $T_i$ should be discussed. Under different conditions, the solution to the distribution is different and the process is complex. Thus, we only study the condition when $N_{m,i} = 0$ tentatively, and then we use the simulation method to find the solution of the residual life distribution model.

In a natural storage process, degradation exceeding the failure threshold will not be repaired due to no inspection during the natural storage period. At the beginning of the next inspection period, the degradation is more than the failure threshold. In this case, the product is failed and there no maintenance occurs. Therefore, specific analysis is needed when $N_{m,i} = 0$.

First, we solve the probability $P(\sup_{s_i+0}(X(T_i + s_i) \geq \omega_2)|X_i, N_{m,i} = 0)$. According to the above analysis, there is

$$P(S_i \leq s_i|X_i, N_{m,i} = 0) = P(\sup_{s_i+0}(X(T_i + s_i) \geq \omega_2)|X_i)$$ (18)

Where

$$T_i + s_i \in ((k - 1)T + (k - 1)d, kT + (k - 1)d), k = 1, 2, \ldots$$ (19)

Next, we solve the probability $P(N_m = 0)$.

When no maintenance is carried out, the degradation amount does not exceed the maintenance threshold in any inspection period before time $T_i + s_i$. According to the independent incremental properties of the Wiener process, the relation of the degradation amounts $X(t)$ and $X(t - 1)$ of two adjacent moments in the inspection period is

$$X(t) - X(t - 1) \sim N(\mu_2, \sigma^2)$$ (20)

The probability is

$$P\left(\left(\int_{-\infty}^{t} X(t) - X(t - 1) \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right) dx\right) < 0\right)$$

This denotes that the difference between two neighbouring moments under the values of $\mu_2$ and $\sigma^2$ estimated by the parameters is small and can be ignored. Thus, degradation in the process can be approximated as a monotonous process. Then, the maximum amount of degradation in an inspection period is the amount of degradation at the last moment of the inspection period. Therefore,

$$P(N_m = 0) = \prod_{k=1}^{km} P(X(t_{k,m}) < \omega_2)$$ (21)
Where

\[ t_{k,m} = kT + kd, \quad k = 1, 2, \ldots, k_m, \quad k_m = \frac{t}{T + d} \]

which is the most recent test period before time \( t \).

When \( N_{m,i} > 0 \), the derivation steps are more complex. In order to simplify the solution, a simulation method is applied to imitate the samples for residual storage life prediction.

4.2. Prediction Steps

The calculation steps of the prediction method in this paper are as follows:

**Step 1** Modeling storage and inspection period.

Define the long-term storage and regular inspection of products, and take into account the determination of degradation parameters \( \mu_1, \mu_2, \sigma \), and maintenance threshold \( M_2 \).

**Step 2** Modeling maintenance period.

Define the conditions and completion effects of the maintenance, and take into account the determination of maintenance parameters \( \mu_m, \sigma_m \), and failure threshold \( M_1 \).

**Step 3** Performing prediction.

Define the entire degradation process of the product, and take into account the determination of the upper limit of the number of maintenances \( k \). Finally, the distribution and point estimation of residual life are obtained.

The schematic diagram of the prediction process is shown in Figure 3.

![Figure 3. The prediction process](image)

5. Calculation Examples

In the simulation method, we simulate the storage process according to the probability distribution formulas we have obtained and sampled the statistics to get the distribution of the residual storage life under the simulation scenario. Then, all samples are averaged to get the point estimation of the residual storage life of the product.
5.1. Calculation of Degradation Model

In this part, we set 1000 samples in an experiment when storage time $T = 500\text{h}$, inspection time $d = 10\text{h}$, failure threshold $M_1 = 300$, maintenance threshold $M_2 = 240$, and upper limit of maintenance times $k = 5$. In the primary experiment, which is referred to as experiment 1, the degradation parameters are $\mu_1 = 0.005$, $\mu_2 = 5$, and $\sigma = 0.5$, and the maintenance parameters are $\mu_m = 50$ and $\sigma_m = 1$. The degradation process of products includes storage, inspection, and maintenance. The degradation amount changes as follows:

![Figure 4. Example of degradation process in experiment 1](image)

From Figure 4, it can be seen that the degradation showed the characteristics of fluctuant rise in general. In the storage period, it changes very little. It significantly increases during the inspection period, which is due to the changes of environmental stress during this period. From time 2030 to 2040, when the degradation exceeds the maintenance threshold $M_2$, the first maintenance and degeneration callback are carried out immediately. At time 4080, the number of repairs $k$ reaches the upper limit, and then the product fails when the degradation amount again reaches the maintenance threshold.

In each experiment, residual life values of all samples are collected. The distributing condition in experiment 1 is shown in Figure 5.

![Figure 5. Residual life distribution in experiment 1](image)

From Figure 5, we can find that the samples concentrate on several values. This is because the degradation in the inspection period is much more significant than that in the storage period, and maintenance times are limited. Therefore, many products fail upon reaching the upper limit of repair times, causing the relatively concentrated sample values of storage life.
5.2. Comparison Experiments

To compare the effect of different degradation parameters, residual storage life estimation $L$ is carried out with different storage parameters $\mu_1, \mu_2, \sigma$, and maintenance parameter $\mu_m$. As $\sigma_m$ is more affected by human factors, we do not discuss its impact on the residual life expectancy.

In experiment 2, we change $\mu_1$ from 0.005 to 0.01, and other parameters remain unchanged compared with experiment 1. This experiment simulates the deterioration of environmental conditions during the storage period, when the drift coefficient $\mu_1$ increases.

In experiment 3, we change $\mu_2$ from 5 to 8, and other parameters remain unchanged compared with experiment 1. This experiment simulates the dramatic changes in environmental conditions during the inspection period, when the drift coefficient $\mu_2$ increases.

In experiment 4, we change $\sigma$ from 0.5 to 1, and other parameters remain unchanged compared with experiment 1. This experiment simulates the active volatility of degradation, when the diffusion coefficient $\sigma$ increases.

In experiment 5, we change $\mu_m$ from 50 to 40, and other parameters remain unchanged compared with experiment 1. This experiment simulates reduction of the maintenance ability during the inspection period, when the repairing drift coefficient $\mu_m$ reduces.

The cumulative distribution functions of all experiments are shown in Figure 6.

![Figure 6. The cumulative distribution functions of residual storage life](image-url)

As shown in Figure 6, in each experiment, the cumulative probability curve seems to be stepped, the reason of which is the same as that of the samples concentrating on several values in Figure 5. In addition, the leftmost curve in them belongs to test 3. The maximum value of residual storage life occurs in test 4 and reaches 7500 hours approximately.

The prediction results and corresponding parameter settings of all experiments are shown in Table 1.

<table>
<thead>
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<th>Experiment number</th>
<th>Degradation parameters</th>
<th>Maintenance parameters</th>
<th>$L$ (h)</th>
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<tr>
<td></td>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
<td>$\sigma$</td>
</tr>
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<td>0.005</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.01</td>
<td>5</td>
<td>0.5</td>
</tr>
<tr>
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<td>8</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.005</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.005</td>
<td>5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

From Table 1, besides the prediction values of residual life, we can find the relationship between parameters and residual life. Comparing experiment 1 and experiment 2, residual life is cut down when $\mu_1$ rises. Comparing experiment 1 and experiment 3, residual life is cut down significantly when $\mu_2$ extends. Comparing experiment 1 and experiment 4, residual life changes little when $\sigma$ changes. Comparing experiment 1 and experiment 5, residual life decreases as $\mu_m$ reduces, which means the effectiveness of repair is essential for a longer residual storage life. After these comparisons, we
can find that the drift coefficient $\mu_2$ influences the residual storage life observably. That is to say, the dramatic changes of environmental conditions during the inspection period is very influential.

6. Conclusions

In this paper, we analyse the multi-stage degradation process model of long storage products that are regularly experimented and repaired, taking full account of the impact of different environmental stresses on the degradation rate of a product.

With the assumption that maintenance can occur at any one moment in the natural inspection process, we consider the product to be regular repaired in the process of storage, so that the storage model can be established. Then, we perform parameters estimation to analyse the influence of storage parameters $\mu_3, \mu_4, \sigma$, and maintenance parameter $\mu_m$. To study the prediction of residual life for the storage of product with regular inspection and preventive maintenance, we look for the expression of residual life. Because of the complexity of the formula, we apply a simulation method to solve the model. To sum up, through simulating 1000 samples in a primary experiment, we calculate the estimated value of the residual life as 5019 hours. Meanwhile, there are four extra experiments that are performed to compare the effect of different parameters. We determine that the dramatic changes of environmental conditions during inspection period influence the residual storage life observably.

Although we discuss preventive maintenance, we focus on multi-stage degradation when considering the amount of degradation of the product to follow the Wiener process. Therefore, this paper provides a way of thinking for the residual life study of long-term storage products considering regular inspection and preventive maintenance to a certain extent.

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