Optimized VMD-Wavelet Packet Threshold Denoising based on Cross-Correlation Analysis

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Abstract

To address the problem that wavelet packet denoising is unable to process signals with strong white noise, an optimized VMD-wavelet packet threshold denoising method based on cross-correlation analysis is proposed. This method combines the advantages of VMD and wavelet packet denoising. By decomposing the noisy signal into several modal components using VMD, the excellent modal components are selected from all modal components according to the cross-correlation analysis based critical correlation coefficient. After that, these excellent modal components are processed using the wavelet packet threshold denoising method. Experimental results show that the proposed method has the advantage of denoising signal with strong white noise, which preserves the effective components of signal, overcomes the blindness of traditional VMD denoising methods and ensures the authenticity of the denoised signal.

Keywords: wavelet packet denoising; variational mode decomposition (vmd); excellent modal component; cross-correlation analysis

(Submitted on June 17, 2018; Revised on July 2, 2018; Accepted on August 11, 2018)

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1. Introduction

It is very important for signal processing and fault diagnosis to effectively denoise strong white noise signals and to guarantee the authenticity of signal after denoising [12]. Variational Mode Decomposition (VMD) was proposed in 2014. This method determines the center frequency and bandwidth of each modal component by iteratively searching for the optimal solution of the variational model in the process of obtaining the decomposition components; thus, it can adaptively realize the signal frequency domain segmentation and the effective separation of the modal components [11]. VMD is a completely non-recursive decomposition model operation, which overcomes the EMD modal aliasing problem, highlights the local features of the signal, shows better noise robustness, and has good sampling effects. It is able to effectively deal with weak signals with strong white noise [1,4].

Due to the advantages of VMD, it is widely used in the field of signal processing. For example, Jianguo Wang [9] used the autocorrelation analysis to optimize the VMD to eliminate the noise of the rolling bearing fault signal and extract the signal characteristics. Jiafu Li [5] used the VMD on the laser detection signal denoising. However, VMD still needs improvement in actual use, especially in the field of signal denoising. Because wavelet packet denoising cannot handle signals with strong white noise, combining VMD and wavelet packet denoising is a new approach to suppressing white noise. However, all existing VMD denoising methods process each modal component, which leads to the denoising signal retaining too many noise components unrelated to the original signal. This causes incomplete denoising, which manifests as the blindness of the traditional VMD denoising method. The research shows that for some signals affected seriously by white noise, some modal components have less or even no correlation with the original signal [6]. Processing this kind of modal components may lead to incomplete signal denoising, namely, unrelated noise components retained in the denoised signal [8].

To overcome these shortcomings of traditional VMD denoising, this paper proposes the concept of excellent modal
component, searches the excellent modal components from all the modal components according to the critical correlation coefficient to form an excellent modal component group, then combines the wavelet packet threshold denoising for the optimal modal component group denoising. The results show that the denoising method can effectively restrain the influence of white noise, and the denoising effect has obvious superiority over other methods.

2. Principle of VMD Algorithm

Variational Mode Decomposition is the solving process of the variational problem; by means of searching the optimal variational model iteratively, the algorithm identifies the center frequency and bandwidth of each modal component and realizes the adaptive subdivision of the frequency domain and each modal component of the signal [3]. If the original input signal is \( f(t) \), the constraint expression of the variational model is shown as Equation (1):

\[
\min \left\{ \sum_{t \in [n]} \left[ \left[ \frac{\hat{f}[t]}{\hat{f}} \right] + \left( j \frac{\omega}{2\pi} \right) u_k(t) \right] e^{-j\omega t} \right\}
\]

where \( \{ u_k \} = \{ u_1, u_2, \ldots, u_k \} \) represents each component after decomposition, \( \{ \omega_k \} = \{ \omega_1, \omega_2, \ldots, \omega_k \} \) represents the central frequency of each component, and \( \sum u_i = f(t) \) is the constraint condition that the sum of all modal components is the original input signal.

The quadratic penalty factor \( \alpha \) and Lagrange multiplication \( \sigma(t) \) operator are introduced to solve the optimal solution of the above variational constraints better, where \( \alpha \) is a sufficiently large positive number that ensures the reconstruction accuracy of signal under the influence of Gaussian noise and \( \sigma(t) \) keeps the rigor of the constraints [7]. The extension expressions are shown in Equation (2):

\[
L(u_k(t), \omega_k(t), \sigma(t)) = \alpha \sum_{t \in [n]} \left[ \left( \frac{\hat{f}[t]}{\hat{f}} \right) + \left( j \frac{\omega}{2\pi} \right) u_k(t) \right] e^{-j\omega t} + \left\| f(t) - \sum_{i \in [n]} u_i(t) \right\|^2 + \left\langle \sigma(t), f(t) - \sum_{i \in [n]} u_i(t) \right\rangle
\]

By means of alternating the direction multiplication operator and updating \( u_k^{+1}(t), \omega_k^{+1}(t), \sigma_k^{+1}(t) \) alternately, the "saddle point" of the extension Lagrange expression is obtained [10]. The implementation steps are summarized as follows:

Step 1. Initialize \( u_k, \omega_k, \sigma_k \) and \( n \).

Step 2. Set \( n = n + 1 \), \( k = k + 1 \) and execute the loop. Terminate the loop when it reaches the preset decomposition layer. The modal component and the center frequency update formula are shown in Equation (3):

\[
\hat{u}_k^{+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i \in [n]} \hat{u}_i(\omega) - \sum_{i \in [n]} \hat{u}_i(\omega) + \frac{\hat{\sigma}(\omega)}{2}}{1 + 2\alpha(\omega - \hat{\omega}_k)^2}
\]

where \( \hat{u}_k(\omega), \hat{f}(\omega), \hat{\sigma}(\omega) \) represents the Fourier transform of \( u_k^+(t), f(t), \sigma^+(t) \) respectively. \( \hat{u}_k^{+1}(\omega) \) is the result of \( \hat{f}(\omega) - \sum_{i \in [n]} \hat{u}_i(\omega) \) through the wiener filter. The algorithm estimates the center frequency according to the power spectrum of each modal component [2], and the expression is shown Equation (4):
\[
\omega_{k}^{n+1} = \frac{\int_{0}^{\infty} \left| \left| u_{k} \right| \right|^{2}}{\int_{0}^{\infty} \left| f(\omega) - \sum_{n} \omega_{n} \right|^{2}}
\]

Step 3. Updating \( \sigma \), the update formula is shown in Equation (5):

\[
\sigma \leftarrow \sigma(\omega) + \tau \left( f(\omega) - \sum_{n} \omega_{n} \right)
\]

Set the precision parameter \( \zeta \), terminate the iteration when \( \sum_{n} \left| u_{k} \right|_{2} \leq \zeta \), otherwise, return to step 2.

3. VMD-Wavelet Packet Threshold Denoising

3.1. Wavelet Packet Threshold Denoising.

Wavelet packet threshold denoising is based on the wavelet packet transform algorithm. Wavelet package is able to further decompose detailed signals, so this method has a good denoising effect on complex signals. The specific steps of wavelet packet threshold denoising can be summarized as follows:

Step 1. Select a wavelet packet basis function and determine the number of decomposition layers, then make wavelet packet transform noise signal to obtain a set of wavelet packet node coefficients \( \omega_{j,i} \) and the noise signal node coefficient \( v_{j,i} \).

Step 2. The node coefficient \( \omega_{j,i} \) in step (1) is processed by threshold, and the node coefficient \( \bar{\omega}_{j,i} \) is obtained while making \( \left| \omega_{j,i} - \bar{\omega}_{j,i} \right| \) the minimum as far as possible. In this paper, the node coefficient is processed using the soft threshold method, and the expression of the soft threshold method is expressed in Equation (6):

\[
\bar{\omega}_{j,i} = \begin{cases} 
\text{sgn}(\omega_{j,i}) \cdot \left| \omega_{j,i} - \lambda \right| & \left| \omega_{j,i} \right| \geq \lambda \\
0 & \left| \omega_{j,i} \right| < \lambda
\end{cases}
\]

where \( \lambda \) is the preset threshold.

Step 3. Reconstruct the processed node coefficient using the wavelet packet reconstruction method and obtain the denoised signal.

3.2. VMD-Wavelet Packet Threshold Denoising

Using wavelet packets cannot effectively restrain the influence of strong white noise on signals. However, as VMD has good noise robustness, it can make up for the disadvantages of wavelet packets. The VMD-wavelet packet denoising method combines the advantages of both methods, and specific denoising steps can be summarized as follows:

Step 1. The noise signal is decomposed by VMD, and several modal components are obtained.
Step 2. Use wavelet packet denoising to process the modal components of step 1.
Step 3. Reconstruct the denoised signal.

Although this method is obviously better than the wavelet packet threshold denoising, it also has problems with setting the optimal decomposition layer number, retaining the non-noise component and incomplete denoising.
4. Optimized VMD-Wavelet Packet Denoising by Cross-Correlation Analysis

4.1. Excellent Modal Component Algorithm

In the correlation analysis of signal, the correlation coefficient can describe the correlation degree of two signals. As for two discrete sequences \( x(j) \) and \( y(j) \), the correlation coefficient is shown in Equation (7):

\[
\rho_{xy} = \frac{\sum_{j=1}^{N} x(j) \cdot y(j)}{\sqrt{\sum_{j=1}^{N} x^2(j) \cdot \sum_{j=1}^{N} y^2(j)}} \tag{7}
\]

where \( |\rho_{xy}| \leq 1 \), \( N \) represents the number of sampling points.

Assuming the original signal \( h(j) \) contains different frequency components, modal components \( \tilde{h}(j) \) are obtained through the variational mode decomposition. According to formula (7), the relationship between the original signal and its modal components can be expressed as Equation (8):

\[
\chi = \frac{\sum_{j=1}^{N} h(j) \cdot \tilde{h}(j)}{\sqrt{E_h \cdot E_{\tilde{h}}}} \tag{8}
\]

where \( E_h = \sum_{j=1}^{N} h^2(j) \), \( E_{\tilde{h}} = \sum_{j=1}^{N} \tilde{h}^2(j) \). Adding white noise \( \xi(j) \) to the original signal, the input signal can be depicted as \( f(j) = h(j) + \xi(j) \). Decomposing signal by variational modes, modal components with noise can be obtained. As white noise has the characteristics of uniform distribution, the following equations can be obtained as Equations (9) ~ (10):

\[
f(j) = \tilde{h}(j) + \tilde{\xi}(j) \tag{9}
\]

\[
\text{SNPR} \sum_{j=1}^{N} \tilde{\xi}^2(j) = \sum_{j=1}^{N} \tilde{h}^2(j) \tag{10}
\]

where SNPR represents signal-to-noise power ratio. \( \tilde{h}(j), \tilde{\xi}(j) \) represent the effective frequency component and the effective noise component of the decomposed signal respectively. The correlation coefficient between the original signal and the modal components of the noisy signal are shown in Equation (11):

\[
\gamma = \frac{\sum_{j=1}^{N} h(j) \cdot f(j)}{\sqrt{\sum_{j=1}^{N} h^2(j) \cdot \sum_{j=1}^{N} f^2(j)}} \tag{11}
\]

Equations (8) ~ (10) are substituted into the formula (11). The correlation coefficient between the noisy signal and its modal component are shown as Equation (12):

\[
\gamma = \frac{\chi \sqrt{E_h \cdot E_{\tilde{h}} + \sum_{j=1}^{N} h(j) \cdot \tilde{\xi}(j)}}{\sqrt{1 + \frac{1}{\text{SNPR}} \left( E_h \cdot E_{\tilde{h}} + 2E_{\tilde{h}} \cdot \sum_{j=1}^{N} \tilde{h}(j) \cdot \tilde{\xi}(j) \right)}} \tag{12}
\]
In order to accurately determine whether the modal component contains the effective component of the original signal, according to the above theoretical derivation and experimental summary, the critical correlation coefficient is proposed as Equation (13):

\[ \delta = \frac{SNPR}{SNPR + 1} \times x \]  
(13)

When \( \gamma \geq \delta \), the modal component is known as the excellent modal component, and all the excellent modal components form an excellent modal component group.

4.2. Optimization of VMD-Wavelet Packet Denoising Algorithm Flow

The flow chart of the optimized VMD-wavelet packet threshold denoising based on the cross-correlation algorithm analysis is shown in Figure 1.

5. Signal Denoising and Result Analysis

To validate the advantages of the proposed denoising algorithm, three denoising methods mentioned above are used to process a sinusoidal superposition signal with a maximum frequency of 10000Hz containing different frequency components. The signal is expressed as Equation (14):

\[ h(t) = 3\sin(1000\cdot \pi \cdot t) + 2.5\sin(2000\cdot \pi \cdot t) + 2\sin(4000\cdot \pi \cdot t) + 1.5\sin(6000\cdot \pi \cdot t) + \sin(10000\cdot \pi \cdot t) + 2\sin(20000\cdot \pi \cdot t) \]  
(14)

In addition, strong white noise \( \xi(t) \) was added to make SNR=0db, and the relationship between SNR and SNPR are shown as Equation (15):
where at the current time step, SNR=1. The power spectrum of the noisy signal is shown in Figure 2.

\[
\text{SNR} = 10 \cdot \log_{10}^{\text{SNPR}}
\]  

(15)

In order to make the comparison reliable, the wavelet packet processing adopts a db4 wavelet base, and the “minimax” soft threshold method is adopted for quantitative processing.

5.1. Wavelet Packet Threshold Denoising Processing

Using the db4 wavelet packet to make wavelet packet denoising, the power spectrum of signals after denoising is shown in Figure 3.

5.2. VMD-Wavelet Packet Threshold Denoising Processing

The VMD decomposition layer was set to 10, and the noise signal was decomposed into 10 modal components. After that, the wavelet packet threshold was used to reconstruct the signal, and the denoising result is obtained. The power spectrum of the denoised signal is shown in Figure 4.
5.3. Optimized VMD-Wavelet Packet Threshold Denoising based on Cross-Correlation Analysis Processing

After decomposing the noisy signal with VMD, ten modal components are obtained. Six excellent modal components, \( u_1, u_2, u_3, u_4, u_5 \) and \( u_6 \), are determined by the optimal modal component algorithm. The values of \( \chi, \delta \) and \( \gamma \) are shown in Table 1.

<table>
<thead>
<tr>
<th>( u )</th>
<th>( u_1 )</th>
<th>( u_2 )</th>
<th>( u_3 )</th>
<th>( u_4 )</th>
<th>( u_5 )</th>
<th>( u_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi )</td>
<td>0.62</td>
<td>0.51</td>
<td>0.41</td>
<td>0.30</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.31</td>
<td>0.26</td>
<td>0.20</td>
<td>0.15</td>
<td>0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.61</td>
<td>0.50</td>
<td>0.40</td>
<td>0.29</td>
<td>0.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>

The power spectrum of the denoised signal is shown in Figure 5.

![Power spectrum of optimized VMD-wavelet packet denoising](image-url)
5.4. Results Analysis

The denoising results of three methods are shown in Table 2.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNPR</td>
<td>1.0221</td>
<td>4.7798</td>
<td>6.4035</td>
</tr>
</tbody>
</table>

Note: Method 1 is the wavelet packet threshold denoising, method 2 is VMD-wavelet packet threshold denoising, and method 3 is the optimized VMD-wavelet packet threshold denoising.

Wavelet packet denoising is not ideal for eliminating strong white noise; the signal-to-noise power ratio only increases by 0.0221. The denoising result of signals containing strong white noise is greatly improved by using the VMD-wavelet packet denoising method. After denoising, the signal-to-noise power ratio is 4.7798. However, a series of unrelated noise components of 4248Hz, 4248Hz and 8740Hz will be retained, leading to incomplete denoising. The cross-correlation analysis based optimized VMD-wavelet packet threshold denoising method has a better denoising effect than the above two methods; signal-to-noise power ratio after denoising is 6.4035. Furthermore, it is able to save effective components completely, which ensures the authenticity of the denoised signal.

6. Conclusion

Compared with the wavelet packet threshold denoising method, the VMD-wavelet packet threshold denoising method has made great breakthroughs on denoising results. However, using threshold processing for each modal component not only increases the amount of calculation but also causes problems with determining the optimal number of decomposition layers. Additionally, the denoising results retain too many irrelevant noise components, leading to incomplete denoising. Through validation of the simulation results, the cross-correlation analysis based optimized VMD-wavelet packet threshold denoising method can effectively process signals polluted by strong white noise. It can not only improve the denoising result but also retain all the effective frequency components, overcoming the blindness of traditional VMD-wavelet packet denoising methods and ensuring the authenticity of the denoised signal. Given the advantages of this approach, it can be extrapolated that it will be widely used in the field of signal processing in the future.

Acknowledgement

This work was supported through grants from the Major National Projects Foundation of China (No. 2016YFC0600906), School of Physics and Electronic Information Engineering, Henan Polytechnic University, and Henan Jinghui Technology Co. Ltd. The authors thank two mentors for their guidance and advice.

References


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