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Gaussian Perturbation Whale Optimization Algorithm based on Nonlinear Strategy

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Abstract

Whale Optimization Algorithm (WOA) is a recently developed swarm intelligence optimization algorithm that has strong global search capability. In this work, considering the deficiency of WOA in a local search mechanism and convergence speed, a Gaussian Perturbation Whale Optimization Algorithm based on Nonlinear Strategy (GWOAN) is introduced. By implementing a nonlinear change strategy on the parameters, the swarm is able to enter the local search process faster and thus improve the local exploitation ability of the algorithm. In a later stage, Gaussian perturbation is performed on the current optimal individuals to enrich the population diversity, avoid premature convergence of the algorithm, and improve the global development capability of the algorithm. The results of the comparison experiment between the GWOAN, WOA, and PSO algorithms show that the accuracy of GWOAN in the selected ten function optimization solutions is significantly higher than that of the comparison algorithms, and its optimization efficiency is also better. Among the ten benchmark functions, four can converge to the theoretical optimal value.

Keywords: whale optimization algorithm; swarm intelligence algorithm; nonlinear strategy; Gaussian perturbation; function optimization

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1. Introduction

Optimization problems are common in many fields such as economic management, transportation, and engineering design. To solve the problems, traditional deterministic optimization methods have the disadvantages of requiring rigorous engineering conditions and having difficulty coping with large-scale optimization problems, while the swarm intelligence optimization algorithms based on animal's group behavior is a more effective method. For example, Dorigo [1], inspired by ant foraging behavior, proposed the algorithm of Ant Colony Optimization (ACO). Kennedy et al. [2] proposed Particle Swarm Optimization (PSO) based on bird predation behavior. Based on how cuckoos were looking for bird nests to place bird eggs and combined with the bird Lévy flying behavior, Yang [3] proposed the algorithm of Cuckoo Search via Lévy flights (CS). Through the simulation of the reasonable division of the gray wolf group for hunting behavior, Mirjalil [4] proposed the Grey Wolf Optimizer (GWO).

In 2016, Mirjalili et al. [5] proposed the Whale Optimization Algorithm (WOA) based on the unique bubble-net hunting strategy of humpback whales by mimicking the whales' predation behavior. WOA has the advantages of having simple principles and being easy to understand. The literature has shown that the WOA's optimization performance is better than that of particle swarm optimization (PSO) and gravity search algorithm (GSA).

WOA has attracted the attention of many scholars both domestically and internationally because of its unique optimization mechanism and good algorithm performance. However, as a new type of swarm intelligence optimization method, WOA's application field has yet to be expanded, and its search performance needs to be improved. [6] combined WOA with the tabu search and effectively solved the quadratic assignment problem. [7] introduced the von Neumann topology into the whale algorithm for fault diagnosis of rolling bearings. [8] proposed an improved WOA based on

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nonlinear factors to solve large-scale optimization problems effectively. [9] combined WOA with the power loss index to achieve optimal renewable resources placement in distribution networks. [10] took the Algerian power system as an example and successfully used WOA to solve the optimal reactive power dispatch problem. In [11], under the multi-objective constraints including minimizing power loss, improving voltage distribution, and minimizing operating costs, WOA is used to effectively implement multiple distributed generator layouts in multi-target radial distribution systems. A cosine control factor and polynomial mutation based WOA (CPWOA) was proposed to improve the accuracy and stability of the algorithm [12]. [13] introduced the chaos theory into the WOA and proposed the chaotic whale optimization algorithm (CWOA), which improved the algorithm's global search. In order to enhance the search performance of the WOA, a hybrid whale optimization algorithm with simulated annealing feature selection was proposed [14]. This paper proposed a Gaussian perturbation whale optimization algorithm based on nonlinear strategy (GWOAN). First, the nonlinear variation strategy was designed to improve the accuracy of the algorithm and accelerate the convergence speed. Secondly, Gaussian perturbation was performed on the current optimal individual in the later stage of the algorithm to ensure the diversity of the population and effectively avoid the premature convergence of the algorithm.

2. Whale Optimization Algorithm

The humpback whale, also known as the big winged whale, is named "Humpback Whale" because its back is slightly raised and shaped like a lute. Humpback whales are big compared to other whales. They are generally more than 12 meters in length and weigh more than 20 tons. They feed mainly on krill and small fish. During predation, humpback whales usually make bubbles under the prey and rush through the bubbles in a circular or 9 shape to feed [15]. Goldbogen et al. [16] used the tag sensor to study the unique bubble-net feeding behavior and found two bubble-related actions, "upward spiral" and "double cycle". In the upward spiral phase, the whale first descends to a position 12 meters below the prey and then makes a spiral motion upward; the double cycle refers to coral cycle and capture cycle. Based on the above-mentioned humpback whale predation strategy, Mirjalili et al. abstracted the whale optimization algorithm (WOA), which consists of three stages: encircle prey, bubble-net attack, and search for prey, and there is a 50% probability to attack or circle the prey during the optimization process.

2.1. Encircling Prey

The position of the whale represents a solution in the search space, and the whale can recognize the prey and encircle it. Since the global optimal position in the search space is not known a priori, the WOA assumes that the current best candidate solution is the whale position that is closest to the prey, and other whales will update their positions according to the position of the best candidate solution. The position update equation is as follows:

$$x(t+1) = x_*(t) - A \cdot D \tag{1}$$

$$D = |C \cdot x_*(t) - x(t)| \tag{2}$$

Where t is the current iteration, x_* is the position vector of the best solution, and Equation (2) shows D is the absolute value of the encircling length. The parameter vectors *A* and *C* are calculated according to the following equations:

$$A = 2 \cdot a \cdot rand_1 - a \tag{3}$$

$$C = 2 \cdot rand_2 \tag{4}$$

In Equation (3) and (4), $rand_1$ and $rand_2$ are random numbers. The update equation for parameter a is as follows:

$$a = 2 - 2 \cdot \frac{t}{t_{max}} \tag{5}$$

In Equation (5), t_{max} is the maximum number of iterations, and the value of a is linearly decreased from 2 to 0.

According to Equation (3), the fluctuation range of the coefficient vector A is [-a, a]. When $|A| < 1$, the whale swims in the shrinking encirclement, in other words, it encircles the prey with a shrinking encircling mechanism.

2.2. Bubble-Net Attack

In order to make the algorithm optimization mechanism conform to the whale's bubble-net feeding behaviour, WOA includes two mechanisms: shrinking encircling mechanism and spiral updating position mechanism. The shrinking encircling mechanism of the algorithm is implemented according to the update of the parameter a. When the coefficient vector $|A| < 1$, the new position of the whale individual can be any position between the original position of the individual and the current best individual position, namely the whale is always swimming in the shrinking encirclement. In the spiral updating position mechanism, the whale attacks the prey in a spiral motion. The position update equation at this time is as follows:

$$x(t+1) = D' \cdot e^{bl} \cdot cos(2\pi l) + x_*(t) \tag{6}$$

$$D' = |x_*(t) - x(t)| \tag{7}$$

In Equation (7), D' represents the distance between the whale and the current optimal individual position, b is a constant 1 that specifies a logarithmic spiral shape, and l is a random vector between [-1, 1]. The humpback whale swims around the prey along a spiral path while swimming in the shrinking encirclement. To simulate this simultaneous behavior, the WOA assumes a 50% probability of choosing a shrinking encircling mechanism or a spiral updating position mechanism.

$$x(t+1) = \begin{cases} x_*(t) - A \cdot D, & if\ p < 0.5 \\ D' \cdot e^{bl} \cdot cos(2\pi l) + x_*(t), & if\ p \geq 0.5 \end{cases} \tag{8}$$

In Equation (8), p is a random number between [0, 1] that represents the probability of selecting to attack or encircle a prey in the process of algorithm optimization.

2.3. Search for Prey

WOA performs a global search based on the change of the parameter vector A. When the coefficient vector $|A| \geq 1$, the whale swims outside the shrinking encirclement and searches randomly. Opposite to the local exploitation stage, the humpback whales at this time search randomly based on each other's position, that is, the algorithm searches for the location update of the individual according to the randomly selected whale individual instead of the current global optimal individual. The whale position updating equation is as follows:

$$x(t+1) = x_{rand}(t) - A \cdot D'' \tag{9}$$

$$D'' = |C \cdot x_{rand}(t) - x(t)| \tag{10}$$

Where x_{rand} represents the position vector of the randomly selected whale individual. In Equation (10), D'' represents the distance between the whale and the randomly selected whale individual position.

3. Improved WOA

The WOA performs global exploration or local exploitation according to the range of the parameter vector A and selects to attack or enclose a prey according to the probability p. To solve WOA's problem of low accuracy and easy premature convergence, this paper first proposes a nonlinear change strategy, which through the nonlinear change of the parameter a, enables the algorithm to enter the local exploitation stage as soon as possible and improves the accuracy of the solution and accelerates the convergence speed. Second, this paper introduces Gaussian perturbation, which is aimed at the current elite individual, in the later stage of the algorithm. This enriches the diversity of the swarm, which helps the algorithm jump out of the local optimum and effectively avoid premature convergence.

3.1. Nonlinear Change Strategy

The whale optimization algorithm selects to perform a global search or a partial search according to the range of the parameter vector *A*. When $|A| < 1$, WOA performs local search according to Equation (1); when $|A| \geq 1$, WOA performs global search according to Equation (9). It can be seen from Equation (3) that the value of the parameter vector *A* is related to the parameter a, that is, when the value of *a* is larger, the algorithm performs global search; when the value of a is smaller, the algorithm performs local search. In order to enhance the local exploitation ability of the algorithm and speed up the convergence, this paper proposes a nonlinear change strategy:

$$a(t) = (a_{ini} - a_{fin}) \cdot (1 - arctan\left(\sqrt{\frac{t}{t_{max}}}\right)) \tag{11}$$

a_{ini} is the initial value of the parameter 2, and a_{fin} is the final value of the parameter 0. The change curve of parameter *a* in the nonlinear change strategy, linear decreasing strategy, and cosine change strategy is shown in Figure 1. In the basic WOA, the parameter *a* is linearly decreased, and the search ability of the algorithm changes linearly, which does not reflect the optimization process of the whale individual well. The cosine variation strategy proposed in [12] makes the parameter *a* slowly decrease in the early stage and rapidly decrease in the later stage. Compared with the basic WOA, the global search ability is improved in the early stage, but the convergence speed is slowed down, and the local exploitation ability is insufficient in the later stage of the algorithm. In the iterative process, the nonlinear change strategy enables the parameter *a* to rapidly decrease in the early stage of the algorithm, so that the whale individual can enter the exploitation stage more quickly, thus improving the accuracy and convergence speed of the algorithm. The change trend of parameter vector *A* under different change strategies is shown in Figure 2. It can be seen clearly that the nonlinear change strategy causes the parameter vector *A* to fall earlier into [-1, 1], that is, the algorithm enters the local exploitation stage earlier, the exploitation ability of the algorithm is enhanced, and the accuracy and speed of the solution is improved.

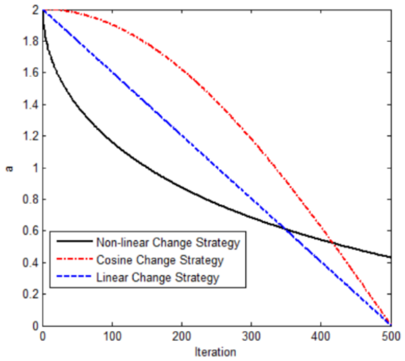
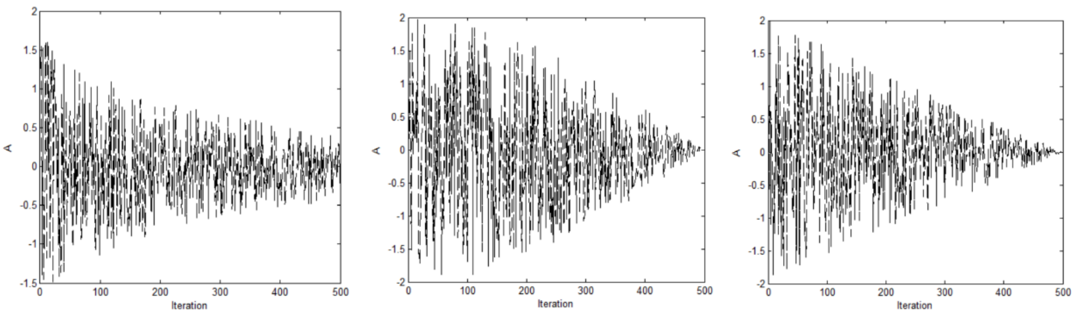


Figure 1. Curve comparison of parameter *a*



(a) Non-linear change strategy (b) Cosine change strategy (c) Linear change strategy

Figure 2. Vector *A* under different change strategies

3.2. Gaussian Perturbation

In the later stage of the iterative process, WOA swarm diversity is reduced and it is easy to converge prematurely.

Performing Gaussian perturbation to the current optimal individual in the later stage of the algorithm can enrich the diversity of the swarm and enhance the ability of the algorithm to jump out of local optimum [17]. The Gaussian distribution curve is symmetrical about the expectation, with the middle high and both ends low, forming a bell shape. Unlike the uniform distribution, the data subject to the Gaussian distribution will be concentrated in some areas. The probability density function is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{(x-\mu)^2}{2\sigma^2}) \tag{12}$$

Through Equation (12), we could see that the bigger the σ, the smaller the probability value of the x position. This explains that the smoother the curve, the more dispersed the probability distribution, while the smaller the σ, the thinner and taller the curve and the more concentrated the probability distribution. According to the characteristics of the distribution curve, the value of μ in the paper is 0, which represents that the peak of the distribution curve is at 0. When the Gaussian perturbation is performed on the current optimal individual in the later stage of the algorithm, the nearby area can be explored to help the algorithm jump out of the local optimum. Figure 3 shows the corresponding three Gaussian distribution curves with different σ values on the condition that $\mu = 0$:

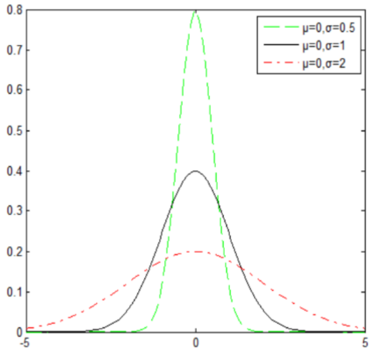


Figure 3. Gaussian distribution curves

From Figure 3, we could see that if the σ value is too large, then the variation range will be too large. Performing Gaussian perturbation at this time will easily lead to algorithm deviation, so it will be difficult to find the global optimal point; however, when the σ value is too small, then the variation range will be too small, and then it cannot achieve the aim of enriching the swarm diversity. Therefore, this paper gives the σ value 1, so as to guarantee that the algorithm can solve effectively and at the same time enrich the swarm diversity and avoid premature convergence. GWOAN proposes the following equation to perform Gaussian perturbation on the optimal individual:

$$x_*(t) = x_*(t) + x_*(t) \cdot normrnd(0,1) \tag{13}$$

Where $normrnd(0,1)$ is a Gaussian distribution function that generates a random number with a mean of 0 and a variance of 1. Performing perturbation on the optimal individual can properly enrich swarm diversity and solve the premature convergence of the algorithm effectively.

3.3. Algorithm Flow

The nonlinear change of parameter a ensures that the algorithm quickly enters the local exploitation stage and increases the accuracy of the solution. Gaussian perturbation on the optimal individual in the later stage of the algorithm can enrich the swarm diversity and effectively avoid the premature convergence of the algorithm. The flow of the GWOAN algorithm is as follows:

Step 1 Set swarm scale *N*, the maximum times of iteration $t_{\max}$, and perform swarm initialization.

Step 2 Calculate the fitness value of each individual and record the current optimal individual and its location.

Step 3 Calculate the value of parameter a according to Equation (11) and update A and C.

Step 4 Generate a random number p between [0,1]. If $p \geq 0.5$, update the individual position according to Equation (6); otherwise, go to step 5.

Step 5 If the parameter vector $|A| < 1$, update the individual position according to Equation (1); otherwise, do so according to Equation (9).-

Step 6 Generate a random number rand. If rand > $(1-t/t_{\max})$, perform Gaussian perturbation on the current optimal individual according to Equation (13); otherwise, go to step 7.

Step 7 Repeat the iterative process from step 2 to step 6 until the set maximum number of iterations is reached.

Step 8 Output the global optimal solution.

4. Numerical Simulation and Analysis

4.1. Benchmark Functions

In this paper, ten benchmark functions are selected [18]. The function name, expression, search range, and theoretical optimal value $f_{\min}$ are shown in Table 1. Among them, F_1-F_5 are unimodal functions, and F_6-F_{10} are multimodal functions. The dimensions of the ten test functions are set to 10, 30, and 100 dimensions respectively.

Table 1. Benchmark functions

Name	Expression	Range	f_{min}
Sphere	$F_1(x) = \sum_{i=1}^{d} x_i^2$	[-100, 100]	0
Schwefel 2.22	$F_2(x) = \sum_{i=1}^{d} \lvert x_i \rvert + \prod_{i=1}^{d} \lvert x_i \rvert$	[-10, 10]	0
Sum squares	$F_3(x) = \sum_{i=1}^{d} i x_i^2$	[-10, 10]	0
Schwefel 2.21	$F_4(x) = max_i\{\lvert x_i \rvert, 1 \leq i \leq n\}$	[-100, 100]	0
Powell Sum	$F_5(x) = \sum_{i=1}^{d} \lvert x_i \rvert^{(i+1)}$	[-1, 1]	0
Zakharov	$F_6(x) = \sum_{i=1}^{d} x_i^2 + (\frac{1}{2}\sum_{i=1}^{d} i x_i)^2 + (\frac{1}{2}\sum_{i=1}^{d} i x_i)^4$	[-10, 10]	0
Ackley	$F_7(x) = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + e$	[-32, 32]	0
Alpine	$F_8(x) = \sum_{i=1}^{d} \lvert x_i \sin(x_i) + 0.1 x_i \rvert$	[-10, 10]	0
Rastrigin	$F_9(x) = \sum_{i=1}^{d} [x_i^2 - 10cos(2\pi x_i) + 10]$	[-5.12, 5.12]	0
Griewank	$F_{10}(x) = \frac{1}{4000}\sum_{i=1}^{d} x_i^2 - \prod_{i=1}^{d}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[-600, 600]	0

4.2. Algorithm Parameter Setting and Experimental Results

In order to verify the performance of the GWOAN proposed in this paper, GWOAN is compared with the basic WOA and the particle swarm optimization algorithm (PSO). The swarm size N for the three algorithms is set to 30, the times of iterations as 500, and the learning factor $c_1 = c_2 = 2$ in the PSO. This paper compares the average precision and standard deviation of the three algorithms after running them independently 30 times to analyse the optimization performance of the three algorithms. The experimental results are shown in Tables 2 to 4.

The experimental simulation environment is MATLAB 2014a, the operating system is Windows10 Home Chinese version, 4.00GB RAM, and the processor is Intel(R) Core(TM) i5-6200U CPU @ 2.30 GHz 2.40GHz.

Table 2. Test results at a dimension of 10

F	GWOAN		WOA		PSO	
	ave	std	ave	std	ave	std
F_1	**6.99E-242**	**0**	9.68E-75	5.29E-74	3.38E-05	6.29E-05
F_2	**8.02E-131**	**3.33E-130**	1.29E-52	3.70E-52	5.76E-02	4.46E-02
F_3	**2.51E-236**	**0**	4.18E-75	2.26E-74	2.14E-04	3.69E-04
F_4	**6.45E-127**	**3.04E-126**	2.54E+00	4.84E+00	4.32E-02	2.65E-02
F_5	**0**	**0**	3.60E-104	1.27E-103	7.99E-09	3.31E-08
F_6	**5.44E-251**	**0**	4.33E+01	3.75E+01	4.31E-03	1.24E-02
F_7	**0**	**0**	3.20E-15	2.35E-15	1.97E-01	4.36E-01
F_8	**3.87E-130**	**1.35E-129**	5.72E-01	1.58E+00	2.67E-02	3.09E-02
F_9	**0**	**0**	2.46E+00	8.24E+00	6.52E+00	6.25E+00
F_{10}	**0**	**0**	6.35E-02	1.32E-01	7.69E-01	3.50E-01

Table 3. Test results at a dimension of 30

F	GWOAN		WOA		PSO	
	ave	std	ave	std	ave	std
F_1	**2.06E-218**	**0**	4.10E-74	1.73E-73	4.22E-01	1.98E-01
F_2	**2.28E-111**	**1.25E-110**	5.01E-51	2.03E-50	3.11E+00	2.69E+00
F_3	**3.23E-221**	**0**	1.81E-73	8.81E-73	1.15E+01	3.65E+01
F_4	**3.05E-111**	**1.67E-110**	3.86E+01	3.02E+01	3.81E+00	2.74E+00
F_5	**0**	**0**	3.77E-103	2.03E-102	3.30E-05	6.21E-05
F_6	**2.31E-228**	**0**	9.36E+02	1.74E+02	7.91E+01	4.28E+01
F_7	**0**	**0**	4.03E-15	2.91E-15	2.30E+00	4.77E-01
F_8	**2.14E-117**	**1.11E-116**	1.83E-41	1.00E-40	2.35E+00	1.26E+00
F_9	**0**	**0**	1.89E-15	1.04E-14	7.27E+01	2.05E+01
F_{10}	**0**	**0**	3.80E-03	2.08E-02	5.45E-02	2.67E-02

Table 4. Test results at a dimension of 100

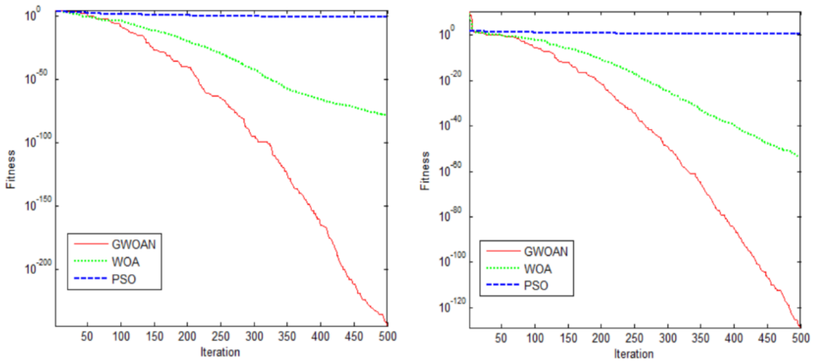
F	GWOAN		WOA		PSO	
	ave	std	ave	std	ave	std
F_1	**1.91E-208**	**0**	2.05E-65	1.12E-64	4.03E+01	9.07E+00
F_2	**4.80E-108**	**1.78E-107**	6.05E-50	2.33E-49	3.18E+01	7.37E+00
F_3	**4.79E-207**	**0**	1.81E-71	5.54E-71	5.14E+02	1.64E+02
F_4	**4.29E-103**	**2.35E-102**	7.72E+01	2.43E+01	1.75E+01	5.42E+00
F_5	**0**	**0**	1.24E-106	5.29E-106	3.69E-04	1.05E-03
F_6	**7.76E-225**	**0**	3.44E+03	2.88E+02	1.47E+03	5.06E+02
F_7	**0**	**0**	3.91E-15	2.35E-15	5.34E+00	8.29E-01
F_8	**1.22E-106**	**6.02E-106**	8.02E-52	5.67E-51	2.71E+01	7.61E+00
F_9	**0**	**0**	**0**	**0**	5.06E+02	5.07E+01
F_{10}	**0**	**0**	9.59E-03	3.29E-02	1.13E+00	1.71E-01

In Tables 2 to 4, the optimal solutions are shown in bold. The unimodal function is often used to test the accuracy of the algorithm. Under the three different dimensions, the solution accuracy of GWOAN for the unimodal functions F_1 and F_3 is over 140 magnitudes higher than that of WOA and over 200 magnitudes higher than that of PSO; the solution accuracy of GWOAN for the unimodal function F_2, is over 50 magnitudes higher than that of WOA and over 100 magnitudes higher than that of PSO; the solution accuracy of GWOAN for the unimodal function F_4 is over 100 magnitudes higher than that of WOA and PSO; and in all 30 independent runs, GWOAN can obtain the theoretical optimal value of the unimodal function F_5 every time, with an obviously better solution performance than the comparison algorithms. Because multimodal functions have a large number of local optima, it is easy to cause premature convergence of the algorithm, so they are often used to test the global optimization ability of the algorithm. Observing the data in the table, we can see that GWOAN can obtain the theoretical optimal value of the multi-peak functions F_7, F_9, and F_{10}. Although GWOAN cannot obtain the theoretical optimal value of F_6 and F_8, the accuracy of GWOAN for F_6 is over 200 magnitudes higher than that of WOA and PSO, and the accuracy for F_8 is over 50 magnitudes higher than WOA and over 100 magnitudes higher than PSO, indicating GWOAN's good performance. In addition, by counting the standard deviation between the 30 running results, except for the functions F_2, F_4, and F_8, the value of the standard deviations of GWOAN for the rest functions is 0, indicating that the GWOAN solution is very stable, that is, the robustness is good. Contrarily, the standard deviations of the comparison algorithms are much higher than those of GWOAN, indicating that the comparison algorithms are unstable when solving the functions. As the dimension increases, the accuracies of the three algorithms all decrease, because the increase of the dimension makes the function solution more complicated.

In summary, among the ten benchmark functions, except obtaining the optimal solution when solving the Function 9 under dimension 100, WOA has a general lower solution accuracy than GWOAN but a higher one compared with PSO. Therefore, the solution performance of WOA ranks in the middle among the three algorithms. PSO failed to obtain the theoretical optimal value in this simulation experiment, and the solution has the lowest accuracy and the worst performance. After 30 independent operations, GWOAN can obtain the theoretical optimal values for four benchmark functions under different dimensions and a much higher solution quality with a solution accuracy closest to the theoretical optimal value for the other six functions compared with WOA and PSO, indicating the effectiveness of the algorithm solution. In addition, GWOAN has stable performance under different dimensions, which indicates that the algorithm is less affected by dimension and has better robustness. Among the three algorithms, GWOAN achieves the best performance.

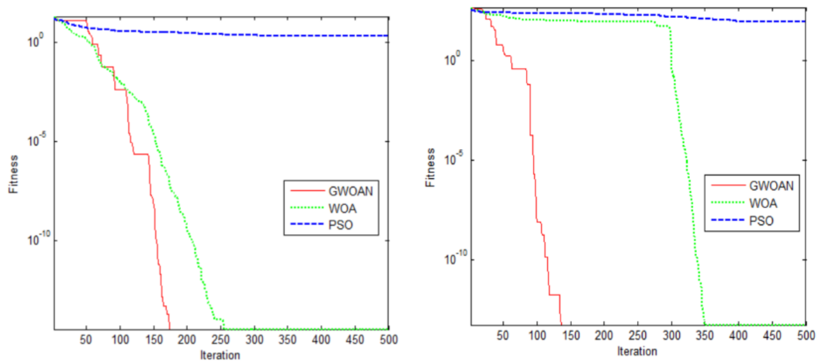
4.3. Analysis of the Convergence Curves

The convergence curve is an important indicator for the performance of the algorithm, through which we can see the convergence speed and the ability of the algorithm to jump out of the local optimum. Due to the limited space, here this paper only gives the convergence curves of the three algorithms when solving four benchmark functions under dimension 30, as shown in Figures 4 and 5.



(a) Convergence curve of Sphere (b) Convergence curve of Schwefel

Figure 4. Convergence curve of the unimodal functions



(a) Convergence curve of Ackley (b) Convergence curve of Rastrigin

Figure 5. Convergence curve of the multimodal functions

The different trends of the three curves in the figures show the difference in the performance of the three algorithms. Figure 4 shows the convergence curve of the unimodal functions. With the iteration, the GWOAN quickly converges globally and continuously improves the accuracy of the solution. The quality of the solution is much higher than that of WOA and PSO. Figure 5 shows the convergence curves of the multimodal functions. The inflection point in the curve shows that the GWOAN algorithm successfully jumps out of the local optimum and continues to optimize, while WOA and PSO converge to the local optimum too early, resulting in a higher curve than GWOAN. In addition, GWOAN can converge to the global optimal solution in 200 iterations, which indicates that the GWOAN algorithm is faster than WOA and PSO. In summary, GWOAN shows stronger optimization performance and higher optimization efficiency.

5. Conclusions

This paper proposes a Gaussian Perturbation Whale Optimization Algorithm based on Nonlinear Strategy (GWOAN). The search ability of the basic WOA varies with the linear variation of the parameters, which cannot reflect the optimization process of the whales well. This work implements the nonlinear perturbation strategy on the parameters to facilitate the algorithm to start the local search process, thus enhancing its ability of exploitation and effectively improving the convergence speed and solution accuracy of the algorithm. In the later stage of the algorithm, as the iteration progresses, the individual follows the optimal individual, thereby reducing the swarm diversity and leading to the premature convergence of the algorithm. This work implements Gaussian perturbation on the optimal individual, which ensures swarm diversity and effectively prevents the algorithm from falling into local optimum. The GWOAN proposed in this paper is compared with the WOA and PSO algorithms. The results of ten benchmark functions show that GWOAN can accurately obtain the theoretical optimal values of the four benchmark functions, and an obviously higher solution accuracy of the other six functions compared with WOA and PSO. GWOAN not only outperforms WOA and PSO in convergence speed and solution accuracy, but also shows better stability and optimization efficiency. Applying the improved WOA to multi-objective combinatorial optimization and practical engineering applications is the focus of the next step.

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