Available online at www.ijpe-online.com

**International Journal of Performability Engineering**

vol. 15, no. 6, June 2019, pp. 1734-1743
DOI: 10.23940/ijpe.19.06.p25.17341743

# Reliability Performance of Improved General Series-Parallel Systems in the Generalized Exponential Lifetime Model

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**Abstract**

Based on the reduction and redundancy methods, the reliability performance of the improved general series-parallel system is considered, assuming the connected components are identically independent and follow the general exponential lifetime model. To extend previous studies, the shape parameter is modified to obtain the reliability equivalence factors of the hot and cold duplications. A hybrid of the hot and cold duplication methods is also considered. Numerical results from a practical example are investigated to illustrate the derived theoretical results of the overall study.

*Keywords*: series-parallel system; generalized exponential distribution; reduction method; hot, cold and hybrid duplications; reliability performance

(Submitted on April 17, 2019; Revised on May 5, 2019; Accepted on May 20, 2019)

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## 1. Introduction

One way to increase the reliability of a system is to use the redundancy method. In this method, standby components are connected in parallel with some or all of the original system components. If the standby components are always switched on with the original components, then we have what is called hot duplication. Cold duplication arises when the standby components are switched off either by perfect or imperfect switch until failure of the original components. Hot and cold duplication methods with different systems configurations were implemented in references [1-7].

Improving the reliability of a system through hot and cold duplication methods has encountered some problems related to space limitation and high manufacturing costs. To overcome such problems, engineers have adopted the reduction method. In this method, the failure rates of some or all of the system components are reduced by a certain factor. When applying this method to increase the reliability of a system, the failure rate of the component should be decreased to achieve the equivalence reliability performance.

The concept of reliability equivalence factor was defined in [8] as a factor by which a characteristic of components of the system design has to be multiplied in order to reach equality of a characteristic for different standard designs. Initially, this concept was applied by [9-10] to improve a simple system using the constant hazard rate of the exponential distribution. The concept was then extensively applied to a variety of the system designs. Studies in [11-13] analyzed the reliability equivalence factors of the basic series and parallel systems. Reliability equivalence factors of abridge network systems and the general parallel series and series parallel systems were addressed in [14-16]. A general parallel system with a mixture of two exponential distributions was also considered in [17].

Beyond the assumptions of constant hazard rates, equivalent studies have been concerned with other lifetime distributions. The gamma distribution was addressed in [18], the Weibull distribution was modeled for a series parallel system in [17], the exponentiated exponential distribution was proposed for the parallel system in [19], reliability

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equivalence factors of a system with a mixture of independent and non-identical components with delay time were obtained in [20], the exponentiated Weibull distribution was considered for a series parallel system in [21], two types of failure rates in a series-parallel systems were analyzed in [22], the reliability equivalence factors of a general parallel system with a mixture of lifetimes were explored in [23], and improving the reliability of a series-parallel system using the modified Weibull distribution was applied in [24].

In most of the pre-frame work, the failure rate functions of a system components mainly increase or decrease through the scale parameter of the proposed lifetime model. In the general reliability analysis, there exist many other lifetime models of which the failure rates are only affected by the shape parameter. This problem was addressed in [25] to test the reliability equivalence factors of a series-parallel system in a Burr-type X distribution. However, a hybrid of hot and cold duplication methods has not been included in reliability research to improve system design.

The generalized exponential distribution introduced in [26] is frequently modeled for lifetime data. It can be used quite effectively to analyze the failure times in place of exponential, Weibull, gamma, and log-normal distributions. The probability density, the reliability, and the hazard rate functions of the generalized exponential distribution are given respectively by

$$f(t) = \left({}^{\delta}/_{\lambda}\right)\left(1-\exp\left({}^{-t}/_{\lambda}\right)\right)^{\delta-1}\exp\left({}^{-t}/_{\lambda}\right),\ t>0 \tag{1}$$

$$R(t) = 1-\left(1-\exp\left({}^{-t}/_{\lambda}\right)\right)^{\delta},\ t>0 \tag{2}$$

$$h(t) = \frac{\left({}^{\delta}/_{\lambda}\right)\left(1-\exp\left({}^{-t}/_{\lambda}\right)\right)^{\delta-1}\exp\left({}^{-t}/_{\lambda}\right)}{1-\left(1-\exp\left({}^{-t}/_{\lambda}\right)\right)^{\delta}},\ t>0 \tag{3}$$

Where $\lambda > 0, \delta > 0$ are the scale and shape parameters, respectively.

For a fixed scale parameter $\lambda$, the hazard rate and the reliability functions are mainly affected by the shape parameter $\delta$. If $\delta > 1$, it has an increasing hazard rate function; if $\delta < 1$, it has a decreasing hazard rate function; and if $\delta = 1$, it coincides with the exponential distribution, which has a constant hazard rate function. More information about the generalized exponential distribution is provided in [27].

The aim of this paper is to evaluate the performance of hot, cold, and a hybrid of hot and cold duplication methods in improving the reliability of a general series parallel system in parallel to the reduction method. The reduction method is carried out by multiplying the shape parameter $\delta$ by a certain factor. The performance criterion will be measured in terms of the reliability equivalence factors and the mean time to failure (*MTTF*) of the improved system.

The rest of this paper is organized as follows. In Section 2, the reliability and the *MTTF* of the series-parallel system are derived. The reduction method and the cold, hot, and hybrid duplication methods for improving the original system are explored in Section 3. The reliability equivalence factors are theoretically formulated in Section 4. Numerical results are conducted through a practical example and listed in Section 4. Finally, Section 5 includes a conclusion of the overall paper.

## 2. General Series-Parallel Systems

The system considered here consists of $m$ subsystems connected in parallel, with subsystem $i$ consisting of $n_i$ components connected in series for $i = 1,2,\cdots,m$. Figure 1 shows the diagram of a series-parallel system.

Let $R_i(t)$ be the reliability of subsystem $i$ and $r_{ij}(t)$ be the reliability of the component $j$ in the subsystem $j = 1,2,\cdots,n_i$, $i = 1,2,\cdots,m$, then

$$R_i(t) = \prod_{j=1}^{n_i} r_{ij}(t) \tag{4}$$

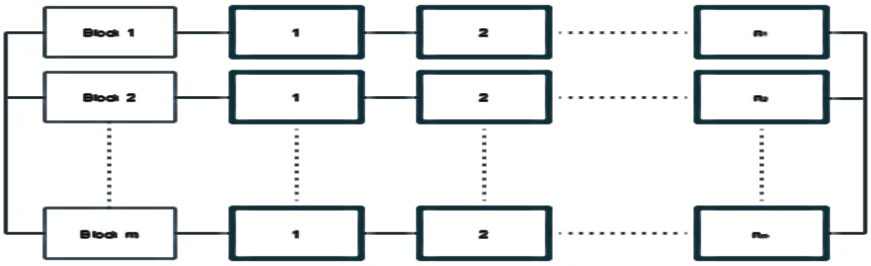


Figure 1. General series-parallel system

This implies that the reliability of the system is

$$R_s(t) = 1 - \prod_{i=1}^{m}(1 - R_i(t)) \tag{5}$$

Using Equation (4), the reliability of the system is

$$R_s(t) = 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{n_i} r_{ij}(t)) \tag{6}$$

Assume that the system components are identically independent and follow the generalized exponential distribution lifetime model. This implies that the reliability of each component is given by

$$r_{ij}(t) = 1 - (1 - \exp\left({}^{t}/_{\lambda}\right))^{\delta}$$

Substituting for $r_{ij}(t)$ in Equation (6), the reliability of the system becomes

$$R_s(t) = 1 - \prod_{i=1}^{m}(1 - \prod_{j=1}^{n_i}(1 - (1 - \exp\left({}^{t}/_{\lambda}\right))^{\delta}) \tag{7}$$

Letting $W(t) = (1 - \exp\left({}^{t}/_{\lambda}\right))^{\delta}$ and simplifying Equation (7), the reliability of the system is given by

$$R_s(t) = 1 - \prod_{i=1}^{m}(1 - (1 - W(t))^{n_i}) \tag{8}$$

Following [22], the *MTTF* for the system is

$$MTTF_s = \int_0^{\infty} R_s(t)\mathrm{d}t \tag{9}$$

## 3. The Improved System

In this section, the reliability function and the *MTTF* of the improved system according to the reduction method and standby redundancy of hot, cold, and hybrid of hot and cold duplication methods will be derived.

### *3.1. The Reduction Method*

In this method, it is assumed that the reliability of $k_i$ components of the subsystem $i, i = 1,2,\cdots,m$ is improved by increasing their reliability function through multiplying the shape parameter $\delta$ by a factor $\rho^d, \rho^d > 0$.

Hence, the reliability of each improved component is

$$r_{RED}(t) = 1 - \left(1 - \exp\left({}^{t}/_{\lambda}\right)\right)^{\rho^{d}\delta} = (1 - W(t)^{\rho^{d}}) \tag{10}$$

This implies that the reliability of the system improved by the reduction method is given by

$$R_{SRED}(t) = 1 - \prod_{i=1}^{m}(1 - (r_{RED}(t))^{k_i}\,(r(t))^{n_i - k_i}) \tag{11}$$

Therefore, the *MTTF* of the system becomes

$$MTTF_{red} = \int_0^\infty R_{SRED}(t)\mathrm{d}t \tag{12}$$

*3.2. The Hot Duplication Method*

In this method, it is assumed that some of the system components are duplicated in parallel and always switched on with the original components. If $h_i$ components of the subsystem $i = 1,2,\cdots,m$ are hot duplicated, then the reliability function of each of these components is given by

$$r_{hot}(t) = r(t)\big(2 - r(t)\big) = (1 - (w(t)^2) \tag{13}$$

This implies that the reliability function of the improved system by the hot duplication method is given by

$$R_{shot} = 1 - \prod_{i=1}^{m}(1 - (r_{hot}(t))^{h_i}\,(r(t))^{n_i - h_i}) \tag{14}$$

Hence, the *MTTF* is

$$MTTF_{hot} = \int_0^\infty R_{shot}\mathrm{d}t \tag{15}$$

*3.3. The Cold Duplication Method*

In this method, it is assumed that some of the system components are connected in parallel with the original components and they are perfectly switched off until failure of the original. Following [8], the reliability function of each component improved by cold duplication via perfect switch is given by

$$r_{cold}(t) = r(t) + \int_0^t f(x)r(t-x)\mathrm{d}x'' \tag{16}$$

Where $f(x)$ is the probability distribution function of the general exponential distribution. If $c_i$ components of the subsystem $i$, $i = 1,2,\cdots,m$ components are cold duplicated via a perfect switch, then the reliability function of the system becomes

$$R_{scold} = 1 - \prod_{i=1}^{m}(1 - (r_{cold}(t))^{c_i}\,(r(t))^{n_i - c_i}) \tag{17}$$

The *MTTF* is

$$MTTF_{cold} = \int_0^\infty R_{scold}\mathrm{d}t \tag{18}$$

### *3.4. The Hybrid Duplication Method*

In this proposed method, we assume that some of the system components are hot duplicated and some are cold duplicated. Suppose $h_i$ components of the subsystem $i$ are hot duplicated and $c_i$ are cold duplicated, $i = 1,2,\cdots,m$. Then, the reliability function of the improved system by a hybrid of these components becomes

$$R_{shyb}(t) = 1 - \prod_{i=1}^{m}(1 - (r_{hot}(t))^{h_i}(r_{cold}(t))^{c_i}\,(r(t))^{n_i-h_i-c_i}) \tag{19}$$

The *MTTF* is

$$MTTF_{hyb} = \int_0^\infty R_{shyb}(t)\mathrm{d}t \tag{20}$$

## 4. Reliability Equivalence Factors

In this section, the reliability equivalence factors of the improved system will be derived. The pre-given reliability level is $\alpha$. The reliability equivalence factor is denoted by $\rho(\alpha)^d$, where $\{d = h, c, hy\}$ presents the hot, cold, and hybrid duplications, respectively. It is defined as a factor by which the shape parameter $\delta$ should be multiplied in order to increase the reliability function of the original system by the reduction method equivalent of improving the system by the hot, cold, and hybrid duplications.

For the hot duplication, $\rho(\alpha)^h$ can be obtained by solving the following set of two equations:

$$R_{SRED}(t) = \alpha,\ R_{shot}(t) = \alpha$$

Substituting for $R_{SRED}(t)$ and $R_{shot}(t)$ from Equations (11) and (14), respectively, $\rho(\alpha)^h$ can be obtained by solving the following two equations simultaneously:

$$\prod_{i=1}^{m}(1 - \left(r_{hot}(t)\right)^{h_i}\,(r(t))^{n_i-h_i}) = 1 - \alpha \tag{21}$$

$$\prod_{i=1}^{m}(1 - \left(r_{RED}(t)\right)^{k_i}\,(r(t))^{n_i-k_i}) = 1 - \alpha \tag{22}$$

Similarly, $\rho(\alpha)^c$ can be obtained by solving the following two equations simultaneously:

$$\prod_{i=1}^{m}(1 - (r_{cold}(t))^{c_i}\,(r(t))^{n_i-c_i}) = 1 - \alpha \tag{23}$$

$$\prod_{i=1}^{m}(1 - \left(r_{RED}(t)\right)^{k_i}\,(r(t))^{n_i-k_i}) = 1 - \alpha \tag{24}$$

$\rho(\alpha)^{hy}$ can also be obtained by solving the following two equations simultaneously:

$$\prod_{i=1}^{m}(1 - (r_{hot}(t))^{h_i}(r_{cold}(t))^{c_i}\,(r(t))^{n_i-h_i-c_i}) = 1 - \alpha \tag{25}$$

$$\prod_{i=1}^{m}(1 - \left(r_{RED}(t)\right)^{k_i}\,(r(t))^{n_i-k_i}) = 1 - \alpha \tag{26}$$

The right-hand sides of the above equations are all polynomials of degree $m$ in $(W(t))^{\rho(\alpha)^d}$, where $W(t) = (1-\exp(^t/_\lambda))^\delta$ and cannot be solved analytically. Therefore, hard computations are needed to obtain $\rho(\alpha)^d$ for the hot, cold, and hybrid duplications. In this paper, we use the Matlab programing language to find solutions for these equations and to evaluate the MTTs of the improved system.

## 5. Numerical Results

To illustrate the theoretical results in the previous sections, we consider a series-parallel system that consists of $n = 6$ components distributed in $m = 2$ subsystems connected in parallel, with $n_1 = 3$ components connected in series in subsystem 1 and $n_2 = 3$ components connected in series in subsystem 2. Assume that the components are identically independent and follow the generalized exponential distribution with fixed scale parameter $\lambda = 2$. The shape parameter $\delta$ is taken to be $\delta$ = 2 to implement an increasing failure rate. The object of this example is to test the reliability performance of the improved system using the hot, cold, and hybrid duplication methods. The corresponding reliability equivalence factors and the *MTTFs* will be the indicators for the improved system using these methods.

The reliability levels are indicated to be $\alpha = 0.2$, $\alpha = 0.5$, and $\alpha = 0.9$. The improved systems are designed by setting

$$(k_1, k_2),(h_1, h_2),(c_1, c_2) = (1,0),(1,1),(2,0),(2,1),(2,2)$$

using the reduction, hot duplication, and cold duplication methods, respectively.

Using the hybrid duplication method, the improved system is considered by the set of configurations: $(c_1, h_1, c_2, h_2) = \{(1,0,0,1),(1,1,0,1),(1,1,1,0),(0,1,1,1),(1,1,1,1)\}$, where $c_1, h_1$ are the numbers of cold and hot duplicated units in subsystem 1 and $c_2, h_2$ are the numbers of cold and hot duplicated units in subsystem 2. The reliability equivalence factors at each setting of hot, cold, and hybrid duplication methods are listed in Tables 1-3. These results are computed using the Matlab programming language. The values of $\rho(\alpha)^d > 5$ mean that it is not possible for engineers to improve a system using the reduction method equivalent to improving a system by hot, cold, and hybrid duplication methods.

The following results are concluded from Tables 1-3:

- For a fixed level of reliability $\alpha$, the reliability equivalence factors decrease as the number of duplicated units decreases using any duplication method. (Tables 1-3)
- As the reliability levels increase, the reliability equivalence factors decrease. This indicates that it is easier for engineers to improve a system through the reduction method with high reliability levels. (Tables 1-3)
- It is not possible to improve a system by the (0,1) reduction method to be equivalent to the improved system by the (1,1), (2,0), (2,1), and (2,2) hot duplication methods. (Table 1)
- It is not possible to improve a system by the (0,1) or (1,1) reduction method to be equivalent to the improved system by the (1,1), (2,0), (2,1), and (2,2) cold duplication methods. (Table 2)
- Reducing the failure rate (increasing the reliability) of a unit by the reliability equivalence factor $\rho(0.20)^h = 3.35$ using the (2,0) reduction method can improve a system as using the (2,2) hot duplication method. (Table 1)
- Reducing the failure rate (increasing the reliability) of a unit by the reliability equivalence factor $\rho(0.90)^c = 1.78$ using the (2,0) reduction method can improve a system as using the (1,1) cold duplication method. (Table 2)
- Reducing the failure rate (increasing the reliability) of a unit by the reliability equivalence factor $\rho(0.50)^h = 2.37$ using the (2,1) reduction method can improve a system as using the (1,1,1,0) hybrid duplication method. (Table 3)
- As the number of improved units increases, the reliability equivalence factors decrease. (This means that less efforts are needed to improve a system by the reduction method). (Tables 1-3)
- For the same configuration of improved units, the cold duplication achieves better performance than the hot and the hybrid duplication methods because it has higher reliability equivalence factors. (Tables 1-3)
- Using the same number of improved units, the hybrid duplication method achieves better performance than the hot duplication method because it has higher reliability equivalence factors. (Tables 1 and 3)
- Improving a system by the (1,1,1,1) hybrid duplication method is equivalent to improving a system by the (2,2) hot and (2,2) cold duplication methods. This is true because the reliability equivalence of this hybrid duplication is equal to the mean of the corresponding reliability equivalence factors using both the (2,2) hot and (2,2) cold duplication methods.
- Unlike the pre-studies, in most of the system configurations, the reliability equivalence factors exceed the value of 1. This is because modification of the reliability function is through the shape parameter and not the scale parameter. (Tables 1-3)

- To overcome the difficulty of improving a system using the reduction method to give equivalent reliability performance as the redundancy method, engineers must reduce the failure rate of more components when using the reduction method. For example, at a level of reliability $\alpha = 0.20$, it is not possible to find $\rho(0.20)^{hy}$ by reduction (1,1) components to be equivalent to hybrid (1,1,1,0) components, but $\rho(0.20)^{hy} = 2.60$ for the (2,1) reduction components. (Table 3)

Table 1. Reliability equivalence factors for the hot duplication method

| Hot duplication | | Reduction | | | | |
|---|---|---|---|---|---|---|
| | $\alpha$ | (0,1) | (1,1) | (2,0) | (2,1) | (2,2) |
| | $\rho(\alpha)^h$ | | | | | |
| (0,1) | 0.20 | 2.00 | 1.51 | 1.39 | 1.26 | 1.20 |
| | 0.50 | 2.00 | 1.42 | 1.33 | 1.24 | 1.17 |
| | 0.90 | 2.00 | 1.31 | 1.26 | 1.18 | 1.15 |
| (1,1) | 0.20 | >5 | 2.00 | 1.76 | 1.54 | 1.38 |
| | 0.50 | >5 | 2.00 | 1.71 | 1.51 | 1.35 |
| | 0.90 | >5 | 2.00 | 1.04 | 1.00 | 0.99 |
| (2,0) | 0.20 | >5 | 2.50 | 1.93 | 1.69 | 1.48 |
| | 0.50 | >5 | 2.31 | 1.89 | 1.60 | 1.42 |
| | 0.90 | >5 | >5 | 1.86 | 1.58 | 1.40 |
| (2,1) | 0.20 | >5 | 3.45 | 2.41 | 2.00 | 1.70 |
| | 0.50 | >5 | 4.85 | 2.39 | 2.00 | 1.68 |
| | 0.90 | >5 | >5 | 2.31 | 2.00 | 1.52 |
| (2,2) | 0.20 | >5 | >5 | 3.35 | 2.52 | 2.00 |
| | 0.50 | >5 | >5 | 3.26 | 2.53 | 2.00 |
| | 0.90 | >5 | >5 | >5 | 3.31 | 2.00 |

Table 2. Reliability equivalence factors for the cold duplication method

| Cold duplication | | Reduction | | | | |
|---|---|---|---|---|---|---|
| | $\alpha$ | (0,1) | (1,1) | (2,0) | (2,1) | (2,2) |
| | | | $\rho(\alpha)^c$ | | | |
| (0,1) | 0.20 | 2.75 | 1.73 | 1.56 | 1.41 | 1.28 |
| | 0.50 | 2.58 | 1.56 | 1.45 | 1.32 | 1.22 |
| | 0.90 | 2.16 | 1.40 | 1.28 | 1.19 | 1.14 |
| (1,1) | 0.20 | >5 | 2.74 | 2.18 | 1.83 | 1.57 |
| | 0.50 | >5 | 2.52 | 2.01 | 1.69 | 1.49 |
| | 0.90 | >5 | 2.19 | 1.78 | 1.48 | 1.33 |
| (2,0) | 0.20 | >5 | >5 | 2.95 | 2.41 | 2.03 |
| | 0.50 | >5 | >5 | 2.71 | 2.14 | 1.73 |
| | 0.90 | >5 | >5 | 2.19 | 1.69 | 1.47 |
| (2,1) | 0.20 | >5 | >5 | 3.74 | 3.01 | 2.33 |
| | 0.50 | >5 | >5 | 3.61 | 2.71 | 2.06 |
| | 0.90 | >5 | >5 | >5 | 2.48 | 1.81 |
| (2,2) | 0.20 | >5 | >5 | >5 | >5 | 3.14 |
| | 0.50 | >5 | >5 | >5 | >5 | 2.75 |
| | 0.90 | >5 | >5 | >5 | >5 | 2.43 |

Table 3. Reliability equivalence factors for the hybrid duplication method

| | | Reduction | | | | |
|---|---|---|---|---|---|---|
| Hybrid | $\alpha$ | (0,1) | (1,1) | (2,0) | (2,1) | (2,2) |
| | | | $\rho(\alpha)^{hyb}$ | | | |
| (0,0,0,1) | 0.20 | >5 | 2.69 | 1.84 | 1.77 | 1.54 |
| | 0.50 | >5 | 2.35 | 1.62 | 1.59 | 1.41 |
| | 0.90 | >5 | 1.90 | 1.56 | 1.39 | 1.27 |
| (1,1,0,1) | 0.20 | >5 | >5 | 2.59 | 2.41 | 1.90 |
| | 0.50 | >5 | >5 | 2.35 | 2.23 | 1.78 |
| | 0.90 | >5 | >5 | 2.07 | 2.39 | 1.69 |
| (1,1,1,0) | 0.20 | >5 | >5 | 2.84 | 2.60 | 2.05 |
| | 0.50 | >5 | >5 | 2.53 | 2.37 | 1.90 |
| | 0.90 | >5 | >5 | 2.34 | 2.21 | 1.67 |
| (0,1,1,1) | 0.20 | >5 | >5 | 3.02 | 2.29 | 1.83 |
| | 0.50 | >5 | >5 | >5 | 2.25 | 1.72 |
| | 0.90 | >5 | >5 | >5 | >5 | >5 |
| (1,1,1,1) | 0.20 | >5 | >5 | >5 | >5 | >5 |
| | 0.50 | >5 | >5 | >5 | >5 | 2.30 |
| | 0.90 | >5 | >5 | >5 | >5 | 2.26 |

**Minitab version 18 package** is used. Figures 2-4 present the reliability of the original system and the improved system by the {hot(1,0), cold(1,0), and hybrid(1,0,0,1)}, {hot(1,1), cold(1,1), and hybrid(1,1,0,1)} and {hot(2,0), cold(2,0), and hybrid(1,1,1,0)} duplications, respectively.

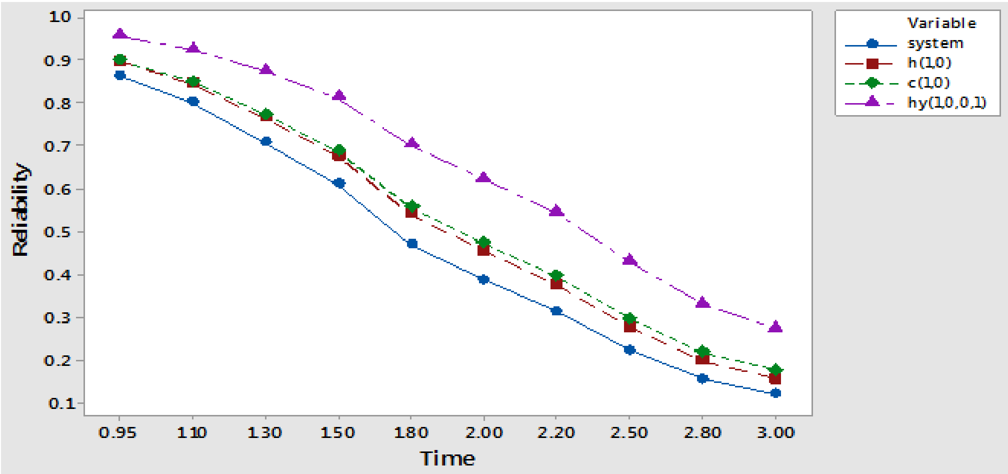


Figure 2. The reliability of the original and the improved systems (hot(1,0),cold(1,0), and hybrid(1,0,0,1))

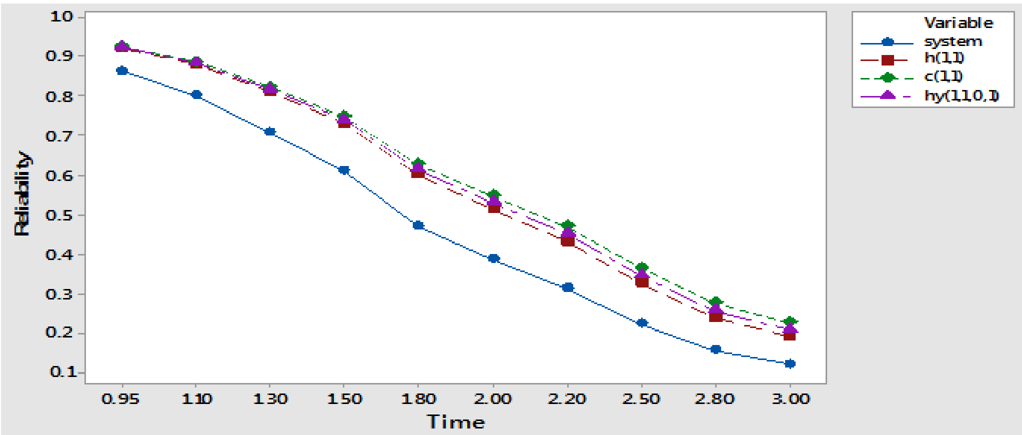


Figure 3. The reliability of the original and the improve systems (hot(1,1), cold(1,1), and hybrid(1,1,0,1))

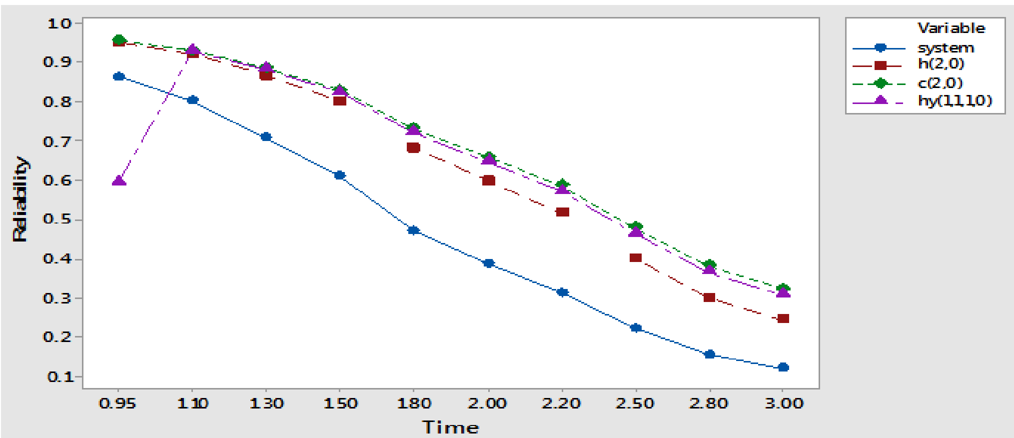


Figure 4. The reliability of the original system and the improve systems (hot(2,0), cold(2,0), and hybrid(1,1,1,0))

Based on these results, we can observe the following:

- Hot duplication of (1,0) components gives an improved design with lowest reliability function among all of the improved designs. (Figures 2-4)

- At a level of reliability $\alpha = 0.61$, the *MTTF* increases from 1.5 time units to 1.60 time units using hot(1,0) components, to 1.72 time units using cold(1,0) components, and to 2.42 time units using hybrid(1,0,0,1) components. (Figure 2)
- At a level of reliability $\alpha = 0.40$, the *MTTF* increases from 1.73 time units to 2.22 time units using hot(1,0) components, to 2.26 time units using cold(1,0) components, and to 2.46 time units using hybrid(1,0,0,1) components. (Figure 2)
- At a level of reliability $\alpha = 0.50$, the *MTTF* increases from 1.70 time units to 2.10 time units using hot(1,1) components, to 2.16 time units using hybrid(1,1,0,1) components, and to 2.20 time units using cold(1,1) components. (Figure 3)
- At a level of reliability $= 0.65$, the improved system by hot(2,0) components has the same *MTTF* = 1.63 time units as the improved system by hybrid(1,1,1,0) components. (Figure 4)
- At a level of reliability $= 0.44$, the improved system by cold(2,0) components has the same *MTTF* = 2.63 time units as the improved system by hybrid(1,1,1,0) components. (Figure 4)
- Generally, with the same number of improved components: $MTTF_{cold} \geq MTTF_{hyb} \geq MTTF_{hot}$. (Figures 2-4)

## 6. Conclusions

In this article, the reliability function and MTTF are investigated to study the reliability performance of the improved series-parallel system using the reduction and redundancy methods. The connected components are assumed to have the generalized exponential lifetime model. Reliability equivalence factors of hot, cold, and hybrid duplication methods are obtained by modifying the shape parameter in the reliability function. Theoretical results are illustrated by a practical example. Numerical results show that, with the same number of improved components, the cold duplication method is the best among other methods. The hybrid duplication method performs better when the number of cold duplicated components increases. In general, the hybrid duplication method achieves better reliability performance for the system than the ordinary hot duplication method. Unlike in previous studies, modification of the shape parameter gives reliability equivalence factors greater than 1 in most of the system configurations. When it is not possible to improve a system by the reduction method, numerical results are assigned by engineers to reduce the failure rate of more components in the system. Results in this paper are restricted for lifetime models that have hazard rate functions mainly affected by the shape parameter. Future studies in this topic may consider the bridge network systems, the non-identical dependent components, and some of the discrete lifetime models with complex systems.

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