Available online at www.ijpe-online.com

International Journal of Performability Engineering

vol. 15, no. 6, June 2019, pp. 1662-1671
DOI: 10.23940/ijpe.19.06.p17.16621671

Reliability Modeling and Analysis of Complex Multi-State Systems based on Weighted Triangular Fuzzy Numbers T-S Fault Tree

Honghua Sun, Hongxia Chen*, and Chunwei Li

School of Mechanical Engineering, Inner Mongolia University of Technology, Hohhot, 010051, China

Abstract

For the problem of T-S fault tree modeling, the probability of failure of the bottom event is not easy to obtain. Firstly, the index is graded, the expert's scoring matrix is obtained according to the scoring standard, the index weight is obtained by applying the entropy method, and the expert weight is obtained by integrating the experts. Secondly, the fuzzy possibility of different fault states of the bottom event is given by experts according to experience and data, and the fuzzy probability of the bottom event is obtained according to the triangular fuzzy number operation rule. The rule execution possibility fuzzy probability of T-S containing triangular fuzzy numbers is derived. Finally, the analysis is carried out in combination with the screw side screw failure of the explosive.

Keywords: complex polymorphic system; T-S fault tree; multi-agent expert; triangular fuzzy number

(Submitted on March 26, 2019; Revised on April 16, 2019; Accepted on June 12, 2019)

1. Introduction

Traditional fault tree analysis methods are based on Boolean algebra and probability theory. The fault tree consists of events and gates. This analysis method has been extensively studied and applied in the reliability, safety, and fault analysis and the diagnosis of complex systems. However, the shortcomings of the traditional fault tree analysis method are reflected in three aspects: (1) the probability of failure of the event needs to be accurately known, (2) the connection between events is precisely known, and (3) the event or component usually has two states: normal or invalid state. These problems sometimes make the fault tree difficult to construct and quantify, limiting the applications of the fault tree. In engineering practice, the failure rate of an event is sometimes difficult to obtain accurately. The fault relationship between events is often uncertain. Events in systems often have multiple fault states.

Huang [1] proposed that a system's uncertainty is divided into two categories: random uncertainty and cognitive uncertainty. Random uncertainty comes from the inherent contingency or variability of the system, which is inevitable. Random uncertainty is often described by probability theory. Cognitive uncertainty is caused by incomplete knowledge and lack of data. Due to the complexity of the system, the limited number of test samples, and the lack of data, the exact values of system state performance levels and state probabilities are usually not available but can be expressed in linguistic form. At this time, the probability-based method is no longer applicable, but non-probability methods, such as the evidence theory [2], fuzzy theory [3], probability box [4], interval theory [5], information difference theory [6], possibility theory [7], and Bayesian method [8], have been proposed and developed for the uncertainty analysis of complex systems.

Song et al. [9] introduced the TS model and fuzzy theory into fault tree analysis. Yao et al. [10] proposed the concept of TS fuzzy importance and its calculation method and used the navigation system as an example to analyze the importance. The validity of the TS fuzzy importance algorithm was verified. Yao et al. [11] gave the TS gate rule form of the Boolean logic gate, verified the feasibility of the TS fault tree, and considered that the Boolean logic gate fault tree is the only TS fault tree. As a special case, the TS fault tree can describe complex system failure mechanisms and polymorphism, and it has better adaptability and flexibility than the Boolean logic gate fault tree. Huang et al. [12] used network analytic

* Corresponding author.
E-mail address: 1990864705@qq.com

hierarchy empowerment of the bottom events. Li et al. [13] used the expert survey method modified by the confidence index to determine the fuzzy possibility of the bottom event. Sun et al. [14] proposed a performance reliability method for analyzing system fault polymorphism by using the ideas expected in statistics. Yao et al. [15] proposed a TS fuzzy fault tree and Bayesian network. The multi-system reliability analysis method can calculate not only the forward reasoning of the system reliability index and importance, but also the reverse reasoning of fault diagnosis. Yao et al. [16] proposed the convex model TS fault tree and importance analysis method. Zhang et al. [17] introduced a hyper-ellipsoid model to describe the bottom event uncertainty.

The above literature applied and studied the T-S fuzzy fault tree. This paper mainly considers the fuzzy uncertainty in the system, uses the triangular fuzzy number to express the fuzzy possibility of the bottom event, gives the normalized processing method of the triangular fuzzy number, and verifies with examples.

2. T-S Fuzzy Fault Tree

Traditional two-state systems often describe the fault states of systems and basic components as normal and faulty states. In practical applications, systems and components often exhibit multiple fault states and fault levels, such as the occurrence of basic events. It does not cause a direct occurrence of a top event system failure, and the system may be in a minor failure phase. In the T-S fuzzy fault tree, the degree of failure is usually expressed by the fuzzy number in the interval [0,1]. If there are no faults, minor faults, or serious faults, the fuzzy numbers 0, 0.5, and 1 can be used respectively. The membership function is a function describing the state of a component in its state domain. Generally, the trapezoidal membership function $\mu(x)$ is used as the membership function of the fuzzy number, and 0, 0.5, and 1 are respectively used as the support set centers of the three fuzzy fault numbers. If the left and right support radius of the membership function is selected as 0.1 and the fuzzy area is selected as 0.3, the membership function diagrams of the three fault states can be obtained as shown in Figure 1.

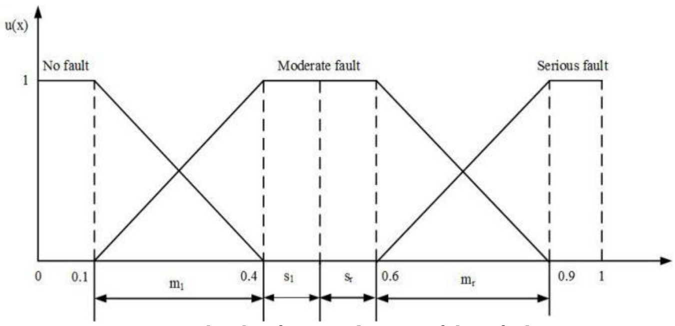

Figure 1. Membership function diagram of three fault states

2.1. T-S Fuzzy Door Rules

The T-S fuzzy model consists of a series of if-then fuzzy rules that use T-S fuzzy gates to describe the associations between events. In the T-S fuzzy fault tree, the bottom event variable is $x_i(i = 1, 2, \cdots, n)$, the intermediate event variable is $y_j(j = 1, 2, \cdots, m)$, and the top event variable is T. Using fuzzy numbers $x_i^{a_i}, y_j^{b_j}, T_q$ describes the fault status of different events separately. Among them, $a_i = 1, 2, \cdots, k_i, b_j = 1, 2, \cdots, l_j,\ q = 1,2\ \ldots, r$. $\mathrm{k_i}$ indicates the fault states number of the bottom event, $\mathrm{l_j}$ indicates the fault states number of the intermediate event, r indicates the fault states number of the top event, respectively. Additionally, $0 \le x_i^1 < x_i^2 < \cdots < x_i^{k_i} \le 1$, $0 \le y_j^1 < y_j^2 < \cdots < y_j^{b_j} \le 1$, $0 \le T_1 < T_2 < \cdots < T_r \le 1$. The fault logic relationship between fault trees is represented by the T-S gate rule. Rule l $(l = 1, 2, \cdots, m)$ indicates the following: for basic event x_1, the fault status is $x_1^{a_1}$; for event x_2, the fault status is $x_2^{a_2}$, ⋯, for event x_n, the fault status is $x_n^{a_n}$; and for intermediate event y, the fault status is y^{b_j}. The possibility is $p^l(y^{b_j})$. s is the total number of rules representing intermediate events, satisfying $s = \prod_{i=1}^{n} k_i$. Assume that the failure probabilities of various fault states of basic events are $P({x_1}^{a_1})$, $P({x_2}^{a_2}), \cdots, P({x_n}^{a_n})$, and then the possibility that the rule l is executed is shown in Equation (1). For superior event y, the fuzzy probability formula is shown in Equation (2). Similarly, when the superior event is the top event, Equation (2) can represent the fuzzy possibility of the top event.

$$p_0^l = P({x_1}^{a_1})P({x_2}^{a_2})P({x_\mathrm{n}}^{a_\mathrm{n}}) \tag{1}$$

$$\begin{cases} P(y^1) = \sum_{l=1}^{s} p_0^l p^l(y^1) \\ P(y^2) = \sum_{l=1}^{s} p_0^l p^l(y^2) \\ \vdots \\ P(y^{b_j}) = \sum_{l=1}^{s} p_0^l p^l(y^{b_j}) \end{cases} \tag{2}$$

2.2. T-S Probability Importance

Importance is a quantitative representation of the importance of each component event in the system. The calculation of importance involves sorting the components of the system in order of their importance in a certain physical sense. It is one of the key parameters for the quantitative analysis of reliability in practical application systems.

Definition 1 When the degree of failure of the component x_j is $x_j^{i_j}$, the probability of being $x_j^{i_j}$ is $P\left(x_j^{i_j}\right)$ $(i_j = 1, 2, \cdots, k_j)$, and the probability that the system top event T is T_q is

$$I_{T_q}^{P_r}\left(x_j^{i_j}\right) = P\left(T_q, P\left(x_j^{i_j}\right) = 1\right) - P\left(T_q, P\left(x_j^{i_j}\right) = 0\right) \tag{3}$$

Where $P\left(T_q, P\left(x_j^{i_j}\right) = 1\right)$ represents the possibility of blurring when the degree of failure of the component x_j is $x_j^{i_j}$. Its fuzzy possibility is $P\left(x_j^{i_j}\right)$. When it is 1, it causes a system top event T for T_q. $P\left(T_q, P\left(x_j^{i_j}\right) = 0\right)$ indicates that the fuzzy probability that the system top event T is T_q when $P\left(x_j^{i_j}\right)$ is 0, causing a system top event T for T_q. The possibility of blurring can be understood as being caused by other degrees of failure T for T_q. Then, $I_{T_q}^{P_r}\left(x_j^{i_j}\right)$ can be considered as the possibility of blurring when a component is x_j and the degree of failure is $x_j^{i_j}$, causing a system top event T for T_q. Replace $\mathrm{P}\left(\mathrm{x}_\mathrm{j}^{\mathrm{i_j}}\right)$ in the $\mathrm{P}\left(\mathrm{T_q}, \mathrm{P}\left(\mathrm{x}_\mathrm{j}^{\mathrm{i_j}}\right) = 1\right)$ with 1. Replace $\mathrm{P}\left(\mathrm{x}_\mathrm{j}^{\mathrm{i_j}}\right)$ in the $\mathrm{P}\left(\mathrm{T_q}, \mathrm{P}\left(\mathrm{x}_\mathrm{j}^{\mathrm{i_j}}\right) = 0\right)$ with 0.

According to the T-S probability importance analysis of the components under different fault degrees, the T-S probability importance of the components is defined as follows.

Definition 2 The T-S probability importance $I_{T_q}^{P_r}(x_j)$ for the component x_j of the system top event T is T_q.

$$I_{T_q}^{P_r}(x_j) = \frac{\sum_{i_j=1}^{k_j{}'} I_{T_q}^{P_r}\left(x_j^{i_j}\right)}{k_j{}'} \tag{4}$$

Where k_j' indicates the degree of failure is non-zero and the number of failures of the j^{th} component. If the fuzzy number 0, 0.5, or 1 is used to indicate the three fault levels, then the corresponding k_j' is 2.

2.3. Key Importance

The fuzzy probability $P\left(x_j^{i_j}\right), (i_j = 1, 2, \cdots, k_j)$ of the failure degree $x_j^{i_j}$ of the component x_j is the T-S critical importance *W* of the system top event T being T_q, where $P(T = T_q)$ represents the probability that the top event T is T_q.

$$I_{T_q}^{C_r}\left(x_j^{i_j}\right) = \frac{P\left(x_j^{i_j}\right) I_{T_q}^{P_r}\left(x_j^{i_j}\right)}{P(T = T_q)} \tag{5}$$

Definition 3 The T-S key importance $I_{T_q}^{C_r}(x_j)$ for the component x_j of the system top event T is T_q.

$$I_{T_q}^{C_r}(x_j) = \sum_{i_j=1}^{k'_j} \frac{I_{T_q}^{C_r}\left(x_j^{i_j}\right)}{k'_j} \tag{6}$$

3. Cognitive Uncertainty T-S Fault Tree Modeling

3.1. Determination of the Weight of Multi-Agent Experts

The above T-S fault tree reliability analysis methods are based on the assumption that the bottom event failure rate is accurate. However, in actual work, the lack of historical data and various working environments make the failure rate ambiguous. The probability of failure of the bottom event often comes from expert experience. Therefore, the accuracy of the expert experience is related to the accuracy of the analysis result. The accuracy of the expert experience is related to the work field, academic background, working years, and title level. If the indicator is a benefit indicator, the higher the score, the more important the expert and the greater the weight. If the indicator is a cost indicator, the higher the score, the less important the expert and the smaller the weight.

Step 1 Establish a decision matrix containing *n* indicators of *m* experts. z_{ij} is the i^{th} expert and the value under the j^{th} indicator.

$$Z = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \cdots & z_{mn} \end{bmatrix} \tag{7}$$

Step 2 Indicators are divided into benefit indicators and cost indicators. The benefit type is marked as I_1, and the cost type is marked as I_2. In order to eliminate the influence of the dimension between indicators, it is necessary to preprocess the data, convert various types of attribute ranges into dimensionless attributes, and make it possible to compare different attributes.

$$d_{ij} = \frac{z_{ij} - z_j^{min}}{z_j^{max} - z_j^{min}}, \ \ j \in I_1 \tag{8}$$

$$d_{ij} = \frac{z_j^{max} - z_{ij}}{z_j^{max} - z_j^{min}}, \ \ j \in I_2 \tag{9}$$

Correct d_{ij} and normalize.

$$p_{ij} = d_{ij} \Big/ \sum_{i=1}^{m} d_{ij} \tag{10}$$

Step 3 Determine the weight of the expert by the entropy weight method.

Entropy was first introduced by the German physicist Clausius in thermodynamics. In thermodynamics, entropy represents the probability of the thermal state of matter. In information theory, the smaller the uncertainty, the smaller the entropy value. The entropy value is large.

The j^{th} entropy of the indicator is

$$h_j = -\sum_{i=1}^{m} p_{ij} l_n p_{ij} \tag{11}$$

Calculate the indicator difference.

$$k_j = 1 - \frac{h_j}{l_n m} \tag{12}$$

Calculate the entropy weight of the indicator.

$$v_j = \frac{K_j}{\sum_{j=1}^{n} K_j} \tag{13}$$

The expert weight is

$$w_i = \sum_{j=1}^{n} v_j y_{ij} \Big/ \sum_{i=1}^{m} \sum_{j=1}^{n} v_j y_{ij} \tag{14}$$

In Equation (14), the weight of the expert is integrated into the matrix Y and determined by information on all indicators in the middle.

3.2. Triangular Fuzzy Processing of Polymorphic Fault Probability of Bottom Event

For the problem that it is difficult for the multi-state event to obtain the exact probability of different states, this paper uses fuzzy numbers to represent the probability of occurrence of multi-state basic events. There are many forms of fuzzy numbers, such as triangle fuzzy numbers, trapezoidal fuzzy numbers, normal fuzzy numbers, and LR type fuzzy numbers. Triangular fuzzy numbers are convenient to process and algebraic operations are easier, so this paper uses triangular fuzzy numbers to represent the probability of event occurrence. Triangular fuzzy numbers are expressed as $\tilde{p} = (p^l, p^m, p^u)$, with p^u and p^l representing the upper and lower bounds of the support, respectively, and $0 < p^l \le p^m \le p^u$, where p^m expresses $\tilde{p}$. The median value of the membership function is shown in Equation (20).

$$\mu_{\tilde{p}}(p) = \begin{cases} \dfrac{p - p^l}{p^m - p^l}, & p^l \le p \le p^m \\ \dfrac{p^u - p}{p^u - p^m}, & p^m \le p \le p^u \\ 0, & \text{Other} \end{cases} \tag{15}$$

Set two triangular fuzzy numbers $\tilde{p}_1 = (p_1^l, p_1^m, p_1^u)$ with $\tilde{p}_2 = (p_2^l, p_2^m, p_2^u)$. The algorithm is as follows:

$$\tilde{p}_1 + \tilde{p}_2 = (p_1^l + p_2^l,\ p_1^m + p_2^m,\ p_1^u + p_2^m) \tag{16}$$

$$\tilde{p}_1 \times \tilde{p}_2 = (p_1^l . p_2^l, p_1^m . p_2^m, p_1^u . p_2^m) \tag{17}$$

$$\frac{1}{\tilde{p}} = (\frac{1}{p^u}, \frac{1}{p^m}, \frac{1}{p^l}) \tag{18}$$

$$k\tilde{p} = (kp^l, kp^m, kp^u) \tag{19}$$

$\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_m$ indicates the judgment result of m experts on the fault degree of a basic event. After the weight of the comprehensive expert, the fault degree of the basic event is still a triangular fuzzy number.

$$\tilde{p}_x = w_1\tilde{p}_1 + w_2\tilde{p}_2 + \cdots + w_m\tilde{p}_m = (\sum_{i=1}^{m} w_i\, p_i^l,\ \sum_{i=1}^{m} w_i\, p_i^m,\ \sum_{i=1}^{m} w_i\, p_i^{ul}) \tag{20}$$

Equation (1) expands to (21), and Equation (2) expands to (22)

$$\begin{aligned} p_0^l &= P(x_1{}^{a_1})P(x_2{}^{a_2}) \cdots P(x_n{}^{a_n}) = P\big(p_{x_1}^{la_1}, p_{x_1}^{la_1}, p_{x_1}^{la_1}\big)P\big(p_{x_2}^{la_2}, p_{x_2}^{la_2}, p_{x_2}^{la_2}\big) \cdots P\big(p_{x_n}^{la_n}, p_{x_n}^{la_n}, p_{x_n}^{la_n}\big) \\ &= (\prod_{i=1}^{n} p_{x_i}^{la_i}, \prod_{i=1}^{n} p_{x_i}^{ma_i}, \prod_{i=1}^{n} p_{x_i}^{ua_i}) \end{aligned} \tag{21}$$

$$\begin{cases} P(y^1) = \sum_{l=1}^{s} (p^l(y^1) \prod_{i=1}^{n} p_{x_i}^{la_i},\ p^l(y^1) \prod_{i=1}^{n} p_{x_i}^{ma_i},\ p^l(y^1) \prod_{i=1}^{n} p_{x_i}^{ua_i}) \\ P(y^2) = \sum_{l=1}^{s} (p^l(y^2) \prod_{i=1}^{n} p_{x_i}^{la_i},\ p^l(y^2) \prod_{i=1}^{n} p_{x_i}^{ma_i},\ p^l(y^2) \prod_{i=1}^{n} p_{x_i}^{ua_i}) \\ \vdots \\ P(y^{b_j}) = \sum_{l=1}^{s} (p^l(y^{b_j}) \prod_{i=1}^{n} p_{x_i}^{la_i},\ p^l(y^{b_j}) \prod_{i=1}^{n} p_{x_i}^{ma_i},\ p^l(y^{b_j}) \prod_{i=1}^{n} p_{x_i}^{ua_i}) \end{cases} \tag{22}$$

After derivation, both the rule execution degree and the fuzzy possibility of the superior event are triangular fuzzy numbers.

3.3. Triangular Fuzzy Number Normalization

Generally speaking, the value of the probability should be in the interval [0,1]. When the upper event of the triangular fuzzy T-S fault tree is calculated, the result of the calculation exceeds the interval [0,1]. This is inconsistent with the actual result, so the data needs to be normalized. With a set of probability data composed of two triangular fuzzy numbers $\tilde{P}_i = (p_i^l, p_i^m, p_i^u)$, $(1 \le i \le N)$, order $t = \sup_{i=1,\cdots,N}(\tilde{P}_i) = \sup_{i=1,\cdots,N}(p_i^l, p_i^m, p_i^u) = \sup(p_i^u)$. Assume $t = p_k^u, (1 \le k \le N)$, corresponding to the first probability data set k. The fuzzy number is $\tilde{P}_k = (p_k^l, p_k^m, p_k^u)$. The normalization factor is defined as

$$w = \frac{1 - p_k^m}{max(1, p_k^u) - p_k^m} \tag{23}$$

Apply the normalization factor w, and normalize the triangular fuzzy numbers. The fuzzy probability $\tilde{P}_i$ is expressed as

$$\vec{P}_i = (p_i^m - w(p_i^m - p_i^l), p_i^m, p_i^m + w(p_i^u - p_i^m)) \tag{24}$$

3.4. Convert Fuzzy Values to Clear Values

In order to calculate the probability importance and key importance, it is necessary to convert the triangular fuzzy number into an accurate value and then calculate it. Common methods of converting fuzzy numbers into exact values include the total integral algorithm, gravity center method, extreme left maximum method, extreme right maximum method, average maximum method, and mean area method. In theory, the center of gravity method is more reasonable, but the calculation is complicated. Since the triangle fuzzy number is used in this paper, the mean area method can be used. The formula is

$$P = \frac{P^L + 2P^M + P^U}{4} \tag{25}$$

4. Case Study

In order to verify the proposed method, only the BCZH-15 heavy ammonium fry vehicle side screw conveying system is selected as the analysis object. The fault tree is analyzed with the "side spiral cylinder lifting fault" as the top event, and the fault tree is built as shown in Figure 2. There is one top event y_2, one intermediate eventy_1, and three bottom events x_1, x_2, x_3 in the fault tree. X_1 indicates that the side spiral hydraulic oil temperature is too high, X_2 indicates that the flow of the side spiral hydraulic pump is insufficient,X_3 represent the side spiral rotation cylinder hydraulic oil leakage, Y_1 indicates that the thrust of the cylinder the side spiral rotation is insufficient, and Y_2 represents a side spiral lift cylinder failure.

(1) Determination of expert weight

Select four indicators. Each index is given different scores according to the level. See Table 1, and select five experts. The scores of the experts are shown in Table 2.

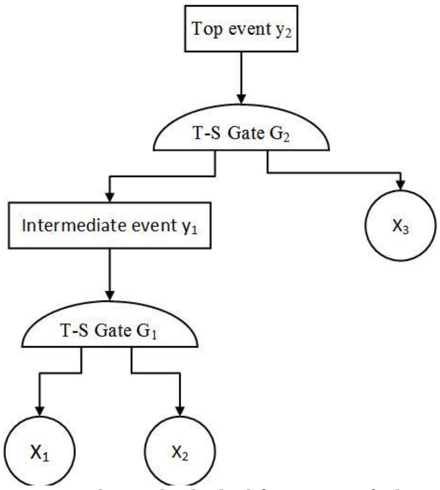

Figure 2. Side spiral cylinder lifting system fault tree

Table 1. Indicators for determining expert weights

Index	Level	Score
Technology	Chief division	5
	System	3
	Component	1
Job title	Researcher and above	5
	Senior engineer	3
	Engineer and below	1
Length of service	More than 20 years	5
	10 to 20 years	3
	Less than 10 years	1
Education	Ph.D. and above	5
	Master's degree	3
	Undergraduate and below	1

Table 2. The results of the five experts scored according to the above table

Expert number	Technology	Job title	Length of service	Education
1	1	3	5	3
2	3	3	3	5
3	5	3	5	3
4	1	3	1	5
5	3	5	5	5

The data in Table 2 calculates the weight of the expert according to Equations (11) to (14). $w_1 = 0.155, w_2 = 0.216, w_3 = 0.219, w_4 = 0.155, w_5 = 0.255$.

(2) Determination of the fuzzy possibility of the bottom event triangle

Due to the lack of historical data, the fuzzy possibility of the bottom event is given by five experts based on experience, and the data of the three bottom events with a triangular blur probability of 1 is shown in Table 3.

Table 3. Bottom event triangle blur possibility

Expert	$x_1(10^{-4})$	$x_2(10^{-4})$	$x_3(10^{-4})$
1	(7, 11, 15)	(12, 15, 18)	(24, 27, 30)
2	(6, 10, 14)	(13, 14, 15)	(23, 25, 27)
3	(6, 8, 10)	(15, 15, 15)	(23, 26, 29)
4	(6, 9, 12)	(14, 16, 18)	(28, 30, 32)
5	(7, 10, 13)	(16, 17, 18)	(25, 28, 31)

Equation (20) is applied to calculate the triangular fuzzy possibility of each bottom event.

$$\tilde{P}_{x_1} = (6.41 \times 10^{-4}, 9.562 \times 10^{-4}, 12.714 \times 10^{-4})$$
$$\tilde{P}_{x_2} = (14.203 \times 10^{-4}, 15.449 \times 10^{-4}, 16.695 \times 10^{-4})$$
$$\tilde{P}_{x_3} = (24.44 \times 10^{-4}, 27.069 \times 10^{-4}, 29.698 \times 10^{-4})$$

(3) Calculation of the fuzzy possibility of the top event from the fuzzy possibility of the bottom event

According to experts and field data, T-S gate rules can be obtained, as shown in Tables 4 and 5.

Table 4. T-S door 1 rule

Rule	x_1	x_2	y_1		
			0	0.5	1
1	0	0	1	0	0
2	0	0.5	0.2	0.3	0.5
3	0	1	0	0	1
4	0.5	0	0.2	0.4	0.4
5	0.5	0.5	0.1	0.3	0.6
6	0.5	1	0	0	1
7	1	0	0	0	1
8	1	0.5	0	0	1
9	1	1	0	0	1

Table 5. T-S door 2 rules

Rule	x_3	y_1	y_2		
			0	0.5	1
1	0	0	1	0	0
2	0	0.5	0.2	0.4	0.4
3	0	1	0	0	1
4	0.5	0	0.2	0.5	0.3
5	0.5	0.5	0.1	0.2	0.7
6	0.5	1	0	0	1
7	1	0	0	0	1
8	1	0.5	0	0	1
9	1	1	0	0	1

Each row in Tables 4 and 5 represents a fuzzy rule, and the rule represented by the fourth row in Table 4 is the following: the degree of failure of x_1 is 0.5. Then, the degree of failure of x_2 is 0. The probability of a failure degree of y_1 is 0.2 when y_1 is 0, the probability of being 0.5 is 0.4, the probability of being 1 is 0.4, and so on. The blur probability of a failure degree of x_1, x_2, x_3 of 0.5 is the same as the blur probability of a failure degree of 1. According to Tables 4 and 5 and Equations (21) and (22), the blurring possibilities of y_1, y_2 are obtained.

$$\tilde{P}_{(y_1=0.5)} = \sum_{l=1}^{9} p_0^l\, p^l(y_1 = 0.5) = (6.821 \times 10^{-4}, 8.454 \times 10^{-4}, 10.086 \times 10^{-4})$$

$$\tilde{P}_{(y_1=1)} = \sum_{l=1}^{9} p_0^l\, p^l(y_1 = 1) = (30.259 \times 10^{-4}, 36.529 \times 10^{-4}, 42.796 \times 10^{-4})$$

$$\tilde{P}_{(y_2=0.5)} = \sum_{l=1}^{9} p_0^l\, p^l(y_2 = 0.5) = (14.893 \times 10^{-4}, 16.842 \times 10^{-4}, 18.786 \times 10^{-4})$$

$$\tilde{P}_{(y_2=1)} = \sum_{l=1}^{9} p_0^l\, p^l(y_2 = 1) = (64.609 \times 10^{-4}, 74.947 \times 10^{-4}, 85.324 \times 10^{-4})$$

(4) T-S probability importance and key importance

The triangular fuzzy probability of the event is converted to the exact value by Equation (25), as shown in Table 6.

Table 6. Probability of the each event

Degree of failure	0.5	1
x_1	9.562×10^{-4}	9.562×10^{-4}
x_2	15.449×10^{-4}	15.449×10^{-4}
x_3	27.069×10^{-4}	27.069×10^{-4}
y_1	8.454×10^{-4}	36.528×10^{-4}
y_2	16.840×10^{-4}	74.956×10^{-4}

Using Equation (3), obtain the T-S probability importance when the failure degree of the bottom event x_1, x_2, x_3 is 0.5 and 1, as shown in Table 7.

Table 7. T-S probability importance of the degree of failure of each bottom event

Probability importance	x_1		x_2		x_3	
	0.5	1	0.5	1	0.5	1
$I_{0.5}^{Pr}\left(x_j^{i_j}\right)$	0.159	0	0.119	0	0.499	0
$I_1^{Pr}\left(x_j^{i_j}\right)$	0.560	1	0.120	1	0.301	1

Using Equation (4), obtain the T-S probability importance of the bottom event x_1, x_2, x_3, as shown in Table 8.

Table 8. T-S probability importance of each bottom event

T-S probability importance	x_1	x_2	x_3
$I_{0.5}^{pr}(x_j)$	0.0795	0.0595	0.2495
$I_1^{pr}(x_j)$	0.78	0.56	0.6505

As can be seen from Table 8, when the system is in a semi-fault and complete failure, the probability importance of x_1 is the greatest.

Combine the fuzzy possibilities of y_1, y_2, which are calculated using Equation (5), to get the critical importance of the T-S bottom event x_1, x_2, x_3 at fault levels of 0.5 and 1, as shown in Table 9.

Table 9. T-S key importance of the degree of failure of each bottom event

Critical importance	x_1		x_2		x_3	
	0.5	1	0.5	1	0.5	1
$I_{0.5}^{Cr}\left(x_j^{i_j}\right)$	0.09	0	0.109	0	0.802	0
$I_1^{Cr}\left(x_j^{i_j}\right)$	0.07 1	0.128	0.025	0.206	0.109	0.361

Using Equation (6), obtain the key importance of the T-S of bottom event x_1, x_2, x_3, as shown in Table 10.

Table 10. T-S probability importance of each bottom event

T-S key importance	x_1	x_2	x_3
$I_{0.5}^{Cr}(x_j)$	0.045	0.0545	0.401
$I_1^{Cr}(x_j)$	0.0995	0.1155	0.235

It can be seen from Table 10 that regardless of what fault state the system is in, the key importance of x_3 is the greatest, the improvement of the hydraulic oil leakage of the side spiral rotating cylinder is the most obvious, and it can be improved by troubleshooting the sequence of x_3, x_2, x_1.The above analysis method can be used in the fault tree of the entire screw conveyor system to provide a basis and method for designing maintenance and troubleshooting.

5. Conclusions

This paper discusses in detail the fuzzy possibility of the multi-agent expert's weight to determine the triangular fuzzy T-S fault tree bottom event when establishing the T-S fuzzy fault tree. It gives a method to determine the fuzzy probability of the bottom event, which effectively processes the engineering system. Due to the ambiguity caused by lack of data or lack of cognition, the T-S fault tree method is more suitable for engineering actual needs. Therefore, the method proposed in this paper has strong engineering practical significance. In this paper, only the case of three fault states is studied, and only some components are calculated. However, the method proposed in this paper is still applicable to the case of more fault states and more complex systems.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (No. 71761030).

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