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International Journal of Performability Engineering

vol. 15, no. 6, June 2019, pp. 1642-1651
DOI: 10.23940/ijpe.19.06.p15.16421651

Complex Network Reliability Analysis based on Entropy Theory

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Abstract

Network reliability is an essential issue of complex networks; the reliability of complex networks plays an important role in the performance in the research process. At the same time, the number of connected nodes in a complex network is a main measure of the complex network. Due to the randomness of complex networks, we define one new degree sequence and the entropy of the complex network, and we then study the entropy of the network as a new measure for the network reliability. The features of entropy are studied in complex networks, and entropy is analyzed in two representative complex network models, the random network model and scale-free network model. The degree distributions functions in the random network model and scale-free network model have significantly different characteristics, the Poisson distribution and Power-law distribution. Furthermore, we study the entropy features under two nodes fault models, random failures and deliberate attacks. We discuss the entropy of the random network model and scale-free network model in two fault modes with the fault intensity gradually increasing from 0 to 1.0. Then, we study the relation between the average degree distribution and the entropy of the network when the fault intensity is 0.3. The results show that the entropy of the network is reasonable to measure the network reliability similar to the number of connected nodes in the network. The purpose of the research is to provide a new way to study network reliability.

Keywords: network reliability; entropy theory; fault intensity; average degree

(Submitted on December 15, 2018; Revised on January 16, 2019; Accepted on February 18, 2019)

1. Introduction

Networks affect every aspect of our lives. Communications networks are made up of telephone and mobile phones, aerospace networks, power networks, and so on. Networks have become an important part of our lives, and the Internet that we use every day can be seen as virtual networks consisting of web pages and hyperlinks. Complex networks greatly facilitate daily life, but they also trouble people due to heavy losses caused by failures; therefore, the reliability of complex networks is a topic that has drawn more and more attention [1-2].

With the fast development of human society, the number of nodes in complex networks continues to increase quickly, and complex networks work in dynamic evolution with complex behavior [3-4]. It seems that the connections between the network nodes are random, but in actuality they follow certain rules. Many linked nodes can be seen as a huge complex network. As the number of nodes in the network increases, the nodes may become faults or the connections between the nodes may fail to work, so the possibility of network failure increases quickly [5]. When many network nodes fail and break off contact with each other, the network is no longer connected, and then the network cannot continue to achieve the required function. The network reliability thus becomes very low [6-7].

Topology structure is an important attribute in complex networks. Small network topology in general is relatively simple, mainly because of its simple structure with less nodes and edges connecting the nodes, and it includes simple star networks, ring networks, and tree networks. The small network reliability with fewer nodes is relatively simple. As the size of the complex networks increases, the network topology structure becomes increasingly complicated, and the traditional indicator of the reliability of complex networks is no longer applicable. It is important to develop new research methods for

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the reliability of complex networks.

Complex networks were first studied by the mathematics of graph theory. They were described as a collection of graphs with nodes and edges, reflecting the essential attributes of the networks. Euler first studied the Seven Bridges of Konigsberg using the graph theory approach, which simplifies the network as a simple mathematical model to derive the actual network complex structural properties [8]. However, because the networks change over time with dynamic evolution, studying complex networks by using graph theory methods has great limitations [9].

After the 1960s, many scholars began studying the development of complex networks mainly through random networks, small-world networks, and scale-free networks. Scale-free networks became a popular modern complex network research model due to their larger cluster coefficient and Power-law degree distribution growth characteristics of these properties with the real-world complex network consistency. The scale-free networks reliability thus became the key research content of complex network research [10-12].

In Section 2 of this paper, we introduce and analyze features of the random network model and scale-free network model through two typical network models, the ER model and BA model. We define one new degree sequence and the entropy for measuring the randomness of the complex network in Section 3. Then, in Section 4, we describe the relation between the new entropy and the average degree in the ER model and BA model, and we explain the different features of the two models. We discuss the entropy under two fault modes with the fault intensity gradually increasing from 0 to 1 in Section 5, and the research shows that the entropy could be used to analyze the network reliability. Then, we study the entropy of the network with an average degree under fault intensity of 0.3 to analyze the network reliability. Finally, we conclude our results regarding network entropy for network reliability research.

2. The Random Network and Scale-Free Network

Random networks and scale-free networks are typical representations of complex networks, and the represented models are the ER model and BA model [10, 12], as shown in Figure 1. For a given complex network, each time a node is removed from the network, all the edges connected to the node are removed, and then a number of other nodes in the network path are also interrupted. If there are multiple paths between two nodes and one path is broken, a path between two nodes could become long, and then the average path length of the entire network could become larger. If all paths between two nodes are interrupted, then there are no connections between the two nodes [13-14]. For a complex network, the average path length and connectivity between nodes are important indicators to measure network reliability. The higher the network connectivity, the smaller the average path length and the greater the ability of the network to complete the required function; therefore, the network has higher reliability [15-16].

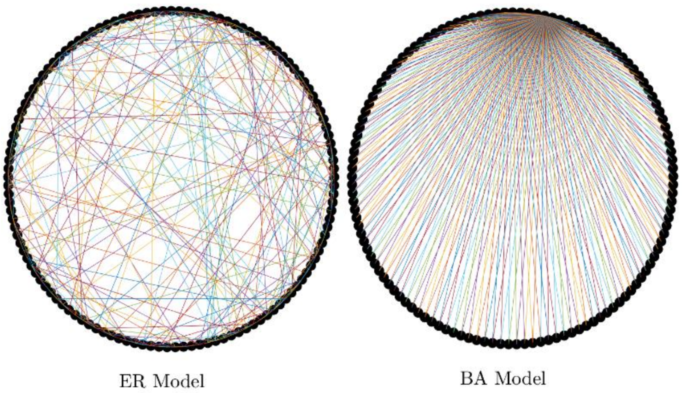

Figure 1. The ER model and BA model

2.1. Random Network

Erdös and Rényi considered that the random network is created by connecting nodes randomly in the network 10, and the amount of nodes is N in the network and the connected probability between two different nodes is p. The total number of edges with N nodes in the network is $\frac{1}{2}N(N-1)$, and M edges are chosen from all the edges to compose a random network.

When the connected probability is p, the average degree of the ER model is:

$$<k>= p(N-1) \approx pN \quad (1)$$

When the number of nodes $N \to +\infty$, the degree distribution function of the ER model is a Poisson distribution:

$$P(k) = C_{N-1}^{k} p^k (1-p)^{N-1-k} \approx \frac{<k>^k e^{-<k>}}{k!} \quad (2)$$

2.2. Scale-Free Network

The random network is a simple ideal network, and the actual complex network usually has the scale-free feature. Most node degrees in the network are small, and little nodes with high degree are called Hub nodes. The whole network shows heterogeneity.

The actual complex networks often have two complex features: growth and preferential attachment. The network initially has a small number of nodes, and by gradually increasing the network nodes and edges, the new nodes connect the existing nodes with Δm edges according to the probability.

Growth and preferential attachment in scale-free networks result in a Power-law degree distribution. The major algorithm of scale-free is as follows 8:

(1) Growth

Starting with m_0 nodes, at each step we add a new network node with m edges ($m < m_0$) that attach old m different nodes that are already present in the network.

(2) Preferential attachment

When choosing the nodes to which the new node connects, we assume that the probability Π that a new network node will be connected to node i depends on the degree k_i of node i, as follows:

$$\Pi = \frac{k_i}{\sum_j k_j} \quad (3)$$

Figure 2 is the evolution process of BA model, the parameter is $\Delta m = m_0 = 2$, and the degree distribution of BA model for sufficiently large networks is in Power-law form:

$$p(k) \sim k^{-q} \quad (4)$$

Where q is the exponent parameter and $2 < q < 3$.

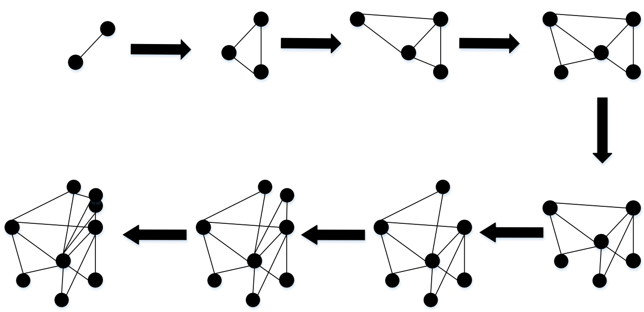

Figure 2. The evolution process of the BA model

Figure 3 shows the degree frequency distribution histogram of the ER model and BA model, and we can find that the ER model is a homogeneous network; however, the BA model possesses significant heterogeneity. Different network topology determines the different reliability of networks, which requires further analysis and research.

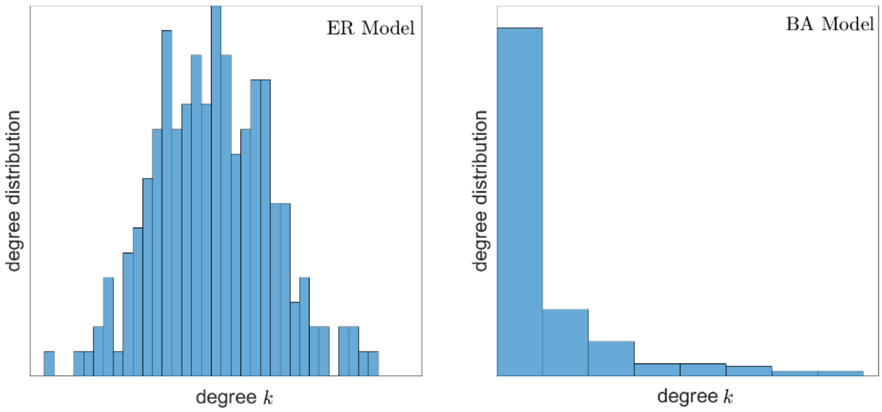

Figure 3. The degree distribution of the ER model and BA model

3. Degree Sequence and the Entropy of the Network

Experts and scholars have conducted in-depth studies on the ER random network model and the scale-free network BA model. Scholars divide faults into two failure modes, random failures and deliberate attacks. Random failures randomly remove a certain percentage of network nodes in a complex network model. Deliberate attacks are in accordance with the size of the node degree. The gradually descending node is removed, and then the network connectivity is analyzed [4, 17-19]. The results shows that the ER model displays obvious vulnerability to failures. Random failures and deliberate attacks cause a decline in network reliability, with deliberate attacks causing a faster rate of decline. The results are shown in Figure 4.

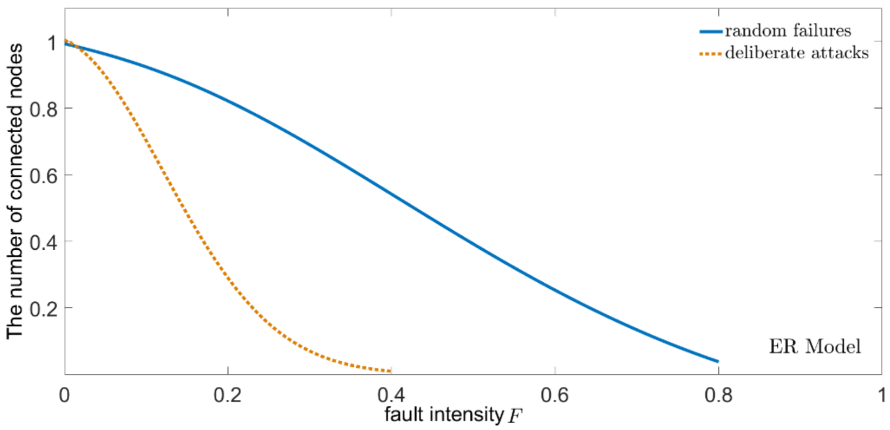

Figure 4. The relation between the number of connected nodes with fault intensity in the ER model

For scale-free networks, the reliability is reduced by random failures, almost nearly 80% of nodes are removed, and the network is no longer connected. However, for deliberate attacks, scale-free networks are very vulnerable. When 20% of the nodes are removed, the network is no longer connected. The results are shown in Figure 5. In the BA model, the network demonstrates good performance and high reliability under the random fault mode. However, for the deliberate attack fault mode, the BA model displays more vulnerability than the ER model does, and when a small number of nodes are removed, the reliability of the BA network becomes very low.

To explain these different characteristics in the ER model and BA model, scholars summarized them for the unevenness of the network. For the ER model, the network is completely uniform, because the degree distribution is the Poisson distribution node, and the degree of most nodes are equal [20-22]. Under the random fault model, by removing most of the nodes, the network becomes disconnected. The BA model is an extremely uneven distribution due to its power law distribution, and most node degrees are small. Therefore, under random failures, the reliability of the BA model is higher 23.

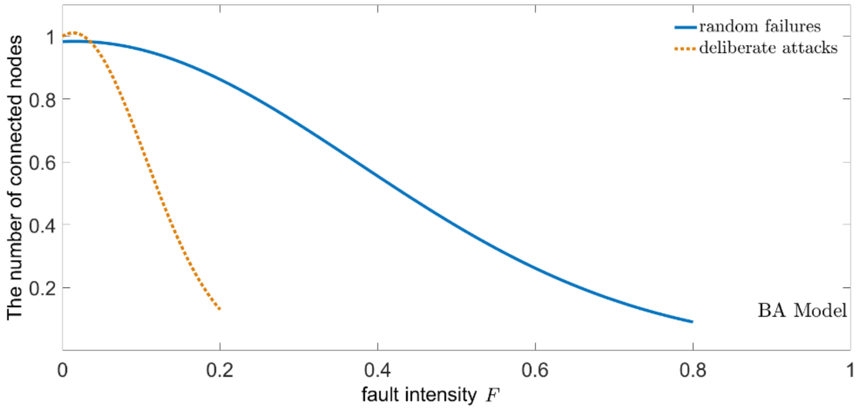

Figure 5. The relation between the number of connected nodes with fault intensity in the BA model

Through the above analysis, we can determine that the reliability of complex networks is correlated to topology inhomogeneity [24-25]. Complex network reliability can be achieved by non-uniformity of the study of complex network analysis. We analyze complex networks from the perspective of the entropy inhomogeneity performed [26-28].

The degree sequence is used to describe network nodes and their degrees. First, we define a degree of sequence; the element is the number of nodes with degrees, and the function is similar to a probability density function. For example, the degree sequence of a network is shown in Figure 6.

$$g=[\frac{2}{10},0,\frac{3}{10},\frac{2}{10},0,\frac{2}{10},\frac{1}{10}] \tag{5}$$

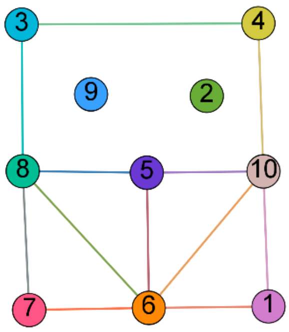
Figure 6. A random network with ten nodes

The entropy $I(G)$ of graph G is a measure of graph structure, or the lack of it. The entropy is the number of bits of "randomness" in a graph. The higher the entropy, the more random the graph. We define the entropy $I(G)$ of graph G as follows [4, 29-30]:

$$I(G)=-\sum_{i=1}^{K}h_i[\ln(h_i)] \tag{6}$$

The entropy $I(G)$ of the network shown in Figure 6 is:

$$I(G)=-\sum_{i=1}^{K}h_i[\ln(h_i)]=-[0.2\times\ln 0.2+0.3\times\ln 0.3+2\times 0.2\times\ln 0.2+0.1\times\ln 0.1]=1.557 \tag{7}$$

For the regular network, the entropy is relatively minimal; this is because the regular network has less randomness. A complete network is shown in Figure 7, and the entropy $I(G)=0$. If the network is an ER model, the entropy is larger with more randomness.

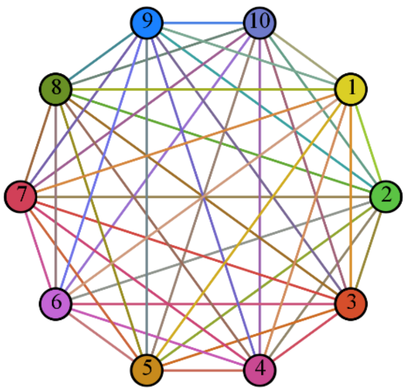

Figure 7. A complete network with ten nodes

4. The Entropy with Average Degree

The ER random network model has two main parameters variables: the total number of network nodes N and the rewiring probability p. The degree distribution of the ER model is a Poisson distribution, so the average degree $<k> \approx Np$. When $N \to +\infty$, we discuss the relation between the entropy $I(G)$ and the average degree $<k>$ in the ER model, and the results are shown in Figure 8.

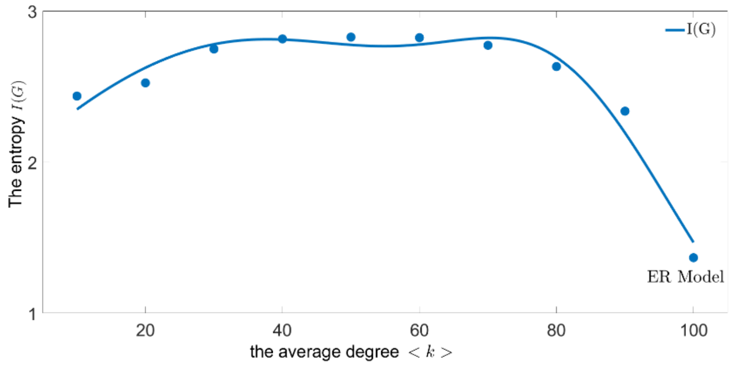

Figure 8. The relation between entropy with the average degree in the ER model

Figure 8 shows the relation on the entropy $I(G)$ and the average degree $<k>$ in the ER model. We can find that when the average degree $<k> \leq 40$, the entropy $I(G)$ increases with the average degree $<k>$, and when the average degree $40 \leq <k> \leq 80$, the entropy $I(G)$ stays constant. When the average degree $<k>$ increases, the entropy $I(G)$ is rapidly reduced. This is because the entropy is the number of bits of "randomness" in the complex network, the entropy $I(G)$ gradually increases with the average degree $<k>$, and the network becomes more and more random. When the average degree $40 \leq <k> \leq 80$, with the restriction of the number of network nodes N, the average degree $<k>$ increases. However, the network randomness stays constant. When the average degree $<k> \geq 80$, the complex network is nearly a complete network and the entropy $I(G)$ is rapidly reduced.

The BA network model has two features, "growth" and "preferential attachment". The BA model has two main parameters variables, the total number of network nodes N and the number of add edges Δm in one step. For the large BA network, the average degree is calculated as follows [31-33]:

$$<k> \approx \frac{2\Delta mt}{N} \approx \frac{2\Delta mt}{t} = 2\Delta m \tag{8}$$

Where t is the number of steps. Then, we discuss the relation between the entropy $I(G)$ and the average degree $<k>$

in the BA model. The results are shown in Figure 9.

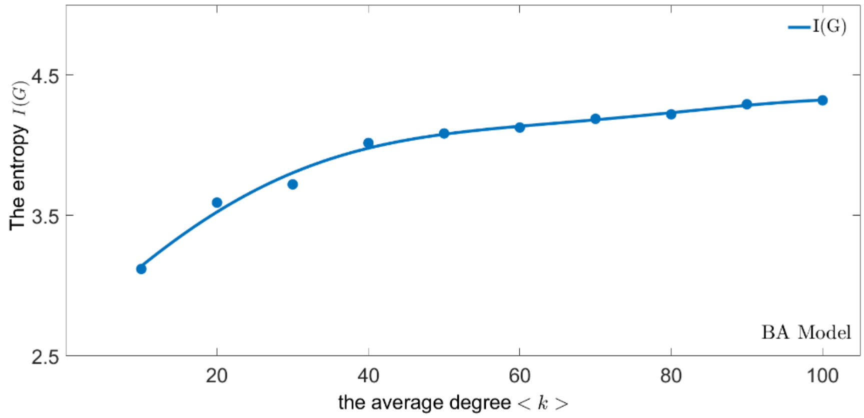

Figure 9. The relation between entropy with the average degree in the BA model

Figure 9 shows the relation on the entropy $I(G)$ and the average degree $< k >$ in the BA model. We can find that when $< k > \leq 40$, the entropy $I(G)$ increases with the average degree $< k >$. When the average degree $< k >$ increases, the entropy $I(G)$ increases slowly. This is because the degree distribution of the BA model is a power law distribution, and the BA model has its own topology structure. The BA model is not completely random. When adding new nodes and edges, the network randomness and entropy $I(G)$ increase. However, the preferential attachment limits the network to keeping its own topology structure, so the entropy $I(G)$ increases slowly when the average degree $< k >$ is nearly 80. At that time, the model topology is stable, so the entropy $I(G)$ is stable too.

5. The Entropy with Fault Intensity

Through the above analysis, we can see that in different failure modes, random failures and deliberate attacks, the network reliability shows different features with different network topology structures. At the same time, the network randomness is measured, so some relation should exist between the network reliability and the network entropy $I(G)$.

Figures 10 and 11 show the relation of the entropy $I(G)$ and the fault intensity F in the ER model and BA model. The results show that the entropy $I(G)$ becomes very small when the fault intensity $F \approx 0.8$ in the two models. For the deliberate attacks, the entropy $I(G)$ is rapidly reduced, and the BA model reduces faster than the ER model does. Then, we can find that the entropy $I(G)$ results have the same features as the number of connected nodes S in Figures 4 and 5, so we believe the entropy $I(G)$ could be used to describe the reliability of the network. A larger entropy $I(G)$ usually indicates a higher network reliability.

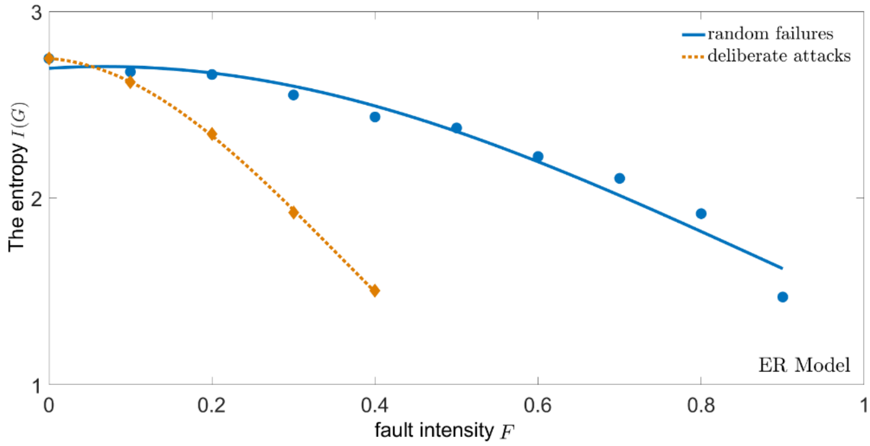

Figure 10. The relation between entropy with fault intensity in the ER model

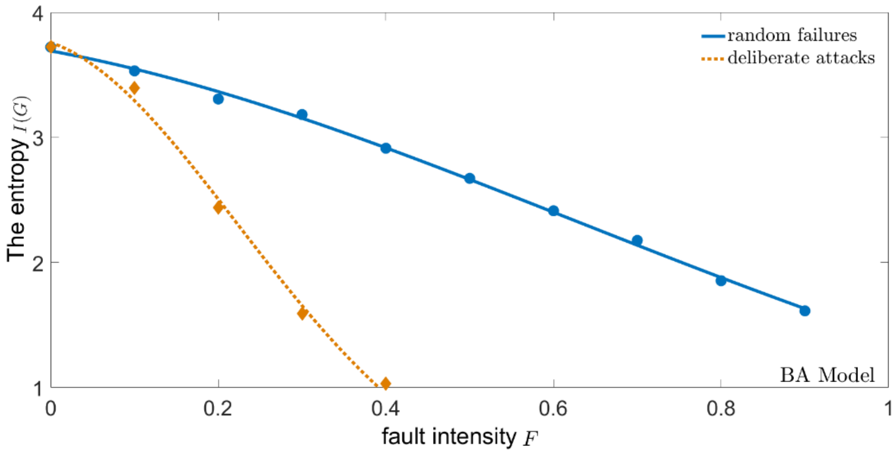

Figure 11. The relation between entropy with fault intensity in the BA model

When the fault intensity $F \approx 0.3$, we study the relation between the entropy $I(G)$ and the average degree $< k >$ in the ER model and BA model, and the results are shown in Figures 12 and 13.

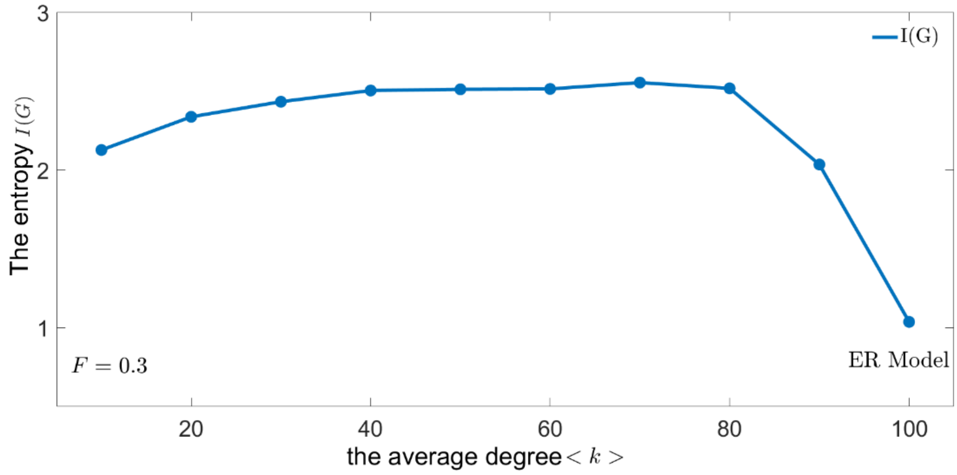

Figure 12. The relation between entropy with the average degree in the ER model

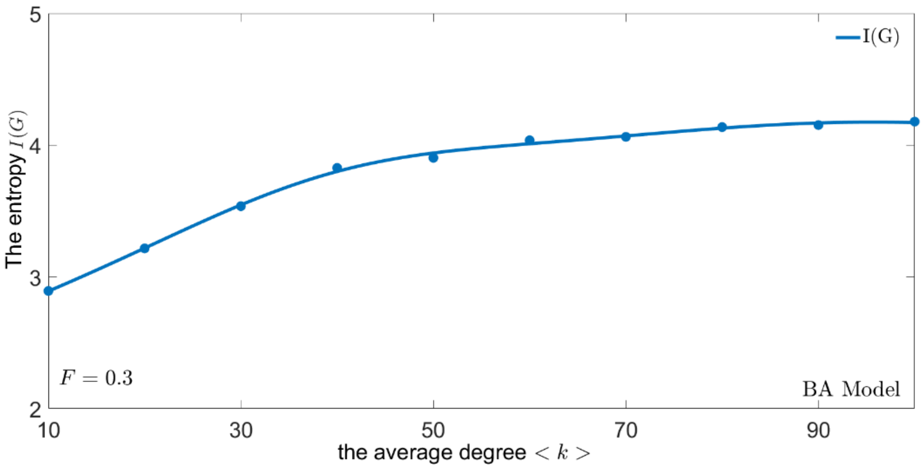

Figure 13. The relation between entropy with the average degree in the BA model

The results in Figures 12 and 13 show that the entropy $I(G)$ has the same features as the results in Figures 8 and 9, but the entropy $I(G)$ in Figures 12 and 13 decreases to some degree with the fault intensity $F = 0.3$. In Figure 12, the entropy $I(G)$ is the largest when the average degree $50 \leq < k > \leq 80$. The ER network reliability is the highest at the same time, because the number of connections between nodes increases quickly as the average degree $<k>$ increases. More displaced edges means higher reliability. For the BA model, the entropy $I(G)$ becomes larger as the randomness of the network increases. More and more nodes and edges are added in this network, so the reliability increases; however, the BA model with its topology structure limits the entropy $I(G)$ and reliability.

6. Conclusions

The construction methods of the random network and scale-free network model are analyzed and summarized in this paper. The analysis results show that the two network models have completely different topological structures, so the reliability features are different under different node faults.

We define the new degree sequence and the entropy to measure the randomness of complex network, and the complete network entropy is 0. At the same time, the network has more randomness, and the entropy is higher.

The entropy in the ER model and BA model have different features. The ER modelHuman Reliabilitys entropy is rapidly reduced when the network nearly becomes a complete network. The BA modelHuman Reliabilitys entropy gradually increases with the average degree, and then the entropy stays constant with the BA network topology structure.

The entropy has the same features as the number of the connected nodes under the fault intensity increase from 0 to 1. The results show that the entropy of the complex network could be used to study the network reliability. When the fault intensity is 0.3, the features of the network show that the fault intensity could reduce the entropy to some degree.

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