

Dynamic Reliability Maintenance for Complex Systems using the Survival Signature

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Abstract

This paper gives a system dynamic reliability maintenance that jointly considers the system residual life and relative importance. Firstly, the component failure time of the system is generated by a Weibull model with unknown shape and scale parameters, and then the two unknown parameters are updated based on the Bayesian rules. The system residual life distribution is estimated by using the theory of survival signature. A novel component relative importance is extended to identify the most critical component groups that need to be maintained. Finally, a system with two cross-linked modules is used to illustrate the usage of our research. Simulation results show that the proposed strategies are effective and convenient.

Keywords: survival signature; Weibull distribution; Bayesian updating; residual life distribution; relative importance index

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Nomenclature

K	The number of component types in the system
m_k	The number of components in type k
$\Phi(l_1, l_2, \dots, l_K)$	The survival signature
C_t^k	The number of type k components function at time t
$P(C_t^k = l_k)$	The predictive probability that there are exactly l_k components of type k function at time t
$T_{k,j}$	The lifetime of j^{th} component of type k in the system
$e_k(e_k^{(t_{\text{now}})})$	The number of type k components that have failed (at time t_{now})
$c_k(c_k^{(t_{\text{now}})})$	The number of type k components that have not failed (at time t_{now})
$t_k(t_k^{(t_{\text{now}})})$	The observation vector of components failure times or censoring times (at time t_{now})
$n_{k,j}, y_{k,j}$	Iteration parameters
c_p	The cost of preventive maintenance
c_c	The cost of corrective maintenance
$T_{\text{sys}}^{(t_{\text{now}})}$	The random system failure time for times $t > t_{\text{now}}$
u	A time span stating from t_{now}
$f_{\text{sys}}^{(t_{\text{now}})}(t)$	The conditional probability density for the (random) system failure time, with support $\{t \in \mathbb{R} : t > t_{\text{now}}\}$
$g^{(t_{\text{now}})}(u)$	The expected operational cycle cost rate
$u_*^{(t_{\text{now}})}$	The cost-optimal time to do maintenance as of t_{now}

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$g_*^{(t_{now})}$

The minimal expected operational cycle cost rate

1. Introduction

In engineering reliability, condition-based maintenance (CBM), also known as predictive maintenance, has attracted more and more attention both in practical applications and academics [1]. Research on CBM is composed of four parts, in which condition monitoring and fault diagnosis are the basis, state prediction is the core procedure, and maintenance decision is the final purpose. This paper focuses on the state predictions of the above four processes.

The state prediction usually has the following difficulties [2]: one is the uncertainty of prediction results, which mainly depends on the randomness of equipment failures and the errors caused by the prediction process itself. The other is the difficulty of data obtaining, whether for the basic data of the equipment operation, the accelerated test data, or the simulation experiment data. Verifying predictive results is difficult as well.

For the state prediction of complex systems, estimating the residual life distribution (RLD) of the system to perform predictive maintenance is a feasible scheme [3]. In practice, it is impossible to perform many system-level tests because of the cost and the test organization. However, the information from testing components is available. However, making full use of the component information to estimate the system residual life distribution is challenging. In [4], the reliability confidence distribution of each component is given, and then the reliability function of the system is obtained based on the structure function. A Bayes-Go method [5] was established to obtain the system reliability by using the component reliability information. In [6], the reliability parameter of component was derived by the Bayes Monte Carlo method, and then the system reliability function was obtained based on system structure. The calculation of the structure function becomes more and more difficult as the complexity of the system structure increases. In recent years, the concept of system signature [7] has been recognized as a popular tool to quantify system reliability, and then RLD can be calculated directly based on the information of the reliability block diagram. The main limitation of system signature is that it assumes that all components in the system belong to the same type. Because complex systems generally contain more than one component type, the survival signature as an alternative to system signature was introduced in [8]. Some recent research and calculation methods can be seen in [9-12]. By means of the survival signature, the RLD (reliability function) $R_{sys}(t)$ of the system can be expressed by the following formula:

$$R_{sys}(t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \Phi(l_1, l_2, \dots, l_K) \prod_{k=1}^K P(C_t^k = l_k) \quad (1)$$

Although many works have been dedicated to system RLD, few have tried to explore the problem of complex system reliability based on the survival signature. This paper studies the problem of the residual life estimation by using the theory of survival signature. In addition, a relative important is proposed, and then a dynamic reliability maintenance plan for a complex system is given based on the system RLD and the important index.

There are two problems that will be solved in this paper. The first one is to predict the system RLD by using the survival signature. The second is to give an appropriate component importance to identify the weaknesses in the current system.

For the first problem, we introduce a novel analytical method where the failure time of the components of the system is generated by a Weibull model with unknown shape and scale parameters. To simplify the Weibull model, many studies assumed that the position parameter is 0 and the shape parameter is a default value [13-14]. However, the wrong choice of the shape parameters will seriously affect the accuracy of the model. In this paper, the shape parameters will be updated together with the scale parameters based on the Bayesian rules. Finally, we can use the survival signature to determine the dynamic RLD for the system. The survival signature can be obtained by using the package of ReliabilityTheory in R [15-16].

For the second problem, we need to determine the weaknesses of the system. The critical components should be the importance information to identify the weaknesses [17]. For the components importance in complex systems, [18] presented three importance indexes, and [19-20] presented generalized importance measures according to Monte Carlo simulation. Most analytical methods are limited to the system with few components, and the traditional simulation method is difficult to calculate. Therefore, we extend a novel component importance introduced from [21] to the component groups. Based on the component groups importance and RLD, system dynamic reliability maintenance is given.

The structure of the rest of the paper is as follows. In Sections 2 and 3, we obtain the expressions of dynamic RLD of the system and the relative importance index (RII) of a specific component or component groups. In Section 4, a system with two cross-linked modules is taken into account to illustrate the usage of our results. Finally, a conclusion and some suggestions for further research are given in Section 5.

2. Bayesian Updating of Weibull Component Models using Right-Censored Observation Data

2.1. Bayesian Update of the Weibull Component Models with Unknown Shape and Scale Parameters

In this study, we assume that the failures of components of different types are independent, and that failures times of components are independent in the same type.

We assume the failure time of the j^{th} component of type k in the system is a Weibull distribution with unknown shape parameter $\beta_k > 0$ and scale parameter λ_k , where $k = 1, \dots, K$. Here, the position parameter is 0, which means the failure can occur any time after the system start-up. Then, the *pdf* (probability density function) and *cdf* (cumulative distribution function) can be rewritten as

$$f(t_{k,j} | \beta_k, \lambda_k) = \frac{\beta_k}{\lambda_k} (t_{k,j})^{\beta_k - 1} e^{-\frac{(t_{k,j})^{\beta_k}}{\lambda_k}} \quad (2)$$

$$F(t_{k,j} | \beta_k, \lambda_k) = 1 - e^{-\frac{(t_{k,j})^{\beta_k}}{\lambda_k}} = P(T_{k,j} \leq t_{k,j} | \beta_k, \lambda_k) \quad (3)$$

Based on β_k and λ_k , the expected lifetime is

$$E[T_{k,j} | \beta_k, \lambda_k] = \lambda_k^{\frac{1}{\beta_k}} \Gamma(1 + \frac{1}{\beta_k}) \quad (4)$$

The prior distribution of the shape parameter is assumed to be the discrete distribution, and the conditional density of the scale parameter is assumed to be the inverse Gamma distribution. The reason why we take the discrete distribution is that we usually know what values the shape parameters can take in some experimental problems, such as for the problem of bearing lifetime. The inverse Gamma distribution is a convenient choice because it is a conjugate prior. Then, the posterior predictive distribution of the unknown parameters can be calculated by adopting the approach described in [22-23]. The prior distribution of shape parameter β_k is s_k finite values in $(0, +\infty)$. Let

$$p'_{k,j} = \Pr(\beta_k = \beta_{k,j}), j = 1, \dots, s_k \quad (5)$$

Where, of course, $\sum_{j=1}^{s_k} p'_{k,j} = 1$. For the scale parameter λ_k , its prior distribution is assumed based on $\beta_k = \beta_{k,j}$. Call it $f(\lambda_k | \beta_{k,j})$, in short, $\lambda_k | \beta_{k,j} \sim IG(a'_{k,j}, b'_{k,j})$:

$$f(\lambda_k | \beta_{k,j}) = f(\lambda_k | a'_{k,j}, b'_{k,j}) = \frac{b'^{a'_{k,j}}_{k,j}}{\Gamma(a'_{k,j})} \lambda_k^{-a'_{k,j}-1} e^{-\frac{b'_{k,j}}{\lambda_k}} \quad (6)$$

For type k components, we assume the number of component lifetimes observations is $m_k = e_k + c_k$. $t_{k,1}, \dots, t_{k,e_k}$ are actual failure observation times for type k components, and $t_{k,1}^+, \dots, t_{k,c_k}^+$ are right-censored observation times for type k components that have not failed. We denote them in an observation vector $t_k = (t_{k,1}, \dots, t_{k,e_k}, t_{k,1}^+, \dots, t_{k,c_k}^+)$, and then the likelihood of this evidence t_k is

$$\begin{aligned}
l(t_k | \beta_{k,j}, \lambda_k) &= [\prod_{i=1}^{e_k} f_{wei}(t_{k,i} | \beta_{k,j}, \lambda_k)] \times [\prod_{i=1}^{c_k} (1 - F_{wei}(t_{k,i}^+ | \beta_{k,j}, \lambda_k))] \\
&= (\frac{\beta_{k,j}}{\lambda_k})^{e_k} [\prod_{i=1}^{e_k} t_{k,i}]^{\beta_{k,j}-1} \exp\{-\frac{\sum_{i=1}^{e_k} (t_{k,i})^{\beta_{k,j}} + \sum_{i=1}^{c_k} (t_{k,i}^+)^{\beta_{k,j}}}{\lambda_k}\}
\end{aligned} \tag{7}$$

Let $u(t_k) = \prod_{i=1}^{e_k} t_{k,i}$, $\tau(t_k) = \sum_{i=1}^{e_k} (t_{k,i})^{\beta_{k,j}} + \sum_{i=1}^{c_k} (t_{k,i}^+)^{\beta_{k,j}}$, and then

$$l(t_k | \beta_{k,j}, \lambda_k) = (\frac{\beta_{k,j}}{\lambda_k})^{e_k} u(t_k)^{\beta_{k,j}-1} e^{-\frac{\tau(t_k)}{\lambda_k}} \tag{8}$$

According to the Bayesian theory, the joint prior distribution $f(\beta_{k,j}, \lambda_k)$ can be updated to the joint posterior distribution $f(\beta_{k,j}, \lambda_k | t_k)$, that is,

$$f(\beta_{k,j}, \lambda_k | t_k) \propto l(t_k | \beta_{k,j}, \lambda_k) \cdot f(\beta_{k,j}, \lambda_k) = p'_{k,j} u(t_k)^{\beta_{k,j}-1} \beta_{k,j}^{e_k} \lambda_k^{-(a'_{k,j}+e_k)-1} \frac{(b'_{k,j})^{a'_{k,j}}}{\Gamma(a'_{k,j})} e^{-\frac{b'_{k,j}+\tau(t_k)}{\lambda_k}} \tag{9}$$

Let $E[\lambda_k | \beta_{k,j}] = y'_{k,j}$. Because of $E[\lambda_k | \beta_{k,j}] = \frac{b'_{k,j}}{a'_{k,j}-1}$, we can use $n'_{k,j}$, $y'_{k,j}$ instead of hyper-parameters $a'_{k,j}$, $b'_{k,j}$ in (9) to make the iteration easier:

$$n'_{k,j} = a'_{k,j} - 1, \quad y'_{k,j} = \frac{b'_{k,j}}{n'_{k,j}} \tag{10}$$

Due to $E[\lambda_k | \beta_{k,j}, n'_{k,j}, y'_{k,j}] = y'_{k,j}$, we can consider $y'_{k,j}$ as the prior guess for the scale parameter λ_k . Using (4), the expected lifetime can be converted λ_k and vice versa.

Referring to the Bayesian update rule in [13, 22], the prior parameters can be updated to posterior parameters:

$$n''_{k,j} = n'_{k,j} + e_k, \quad y''_{k,j} = \frac{n'_{k,j}}{n'_{k,j} + e_k} y'_{k,j} + \frac{e_k}{n'_{k,j} + e_k} \frac{\tau(t_k)}{e_k} \tag{11}$$

The posterior distribution of λ_k is given by

$$\begin{aligned}
\lambda_k | \beta_{k,j} &\sim IG(a''_{k,j}, b''_{k,j}) \\
&\sim IG(n'_{k,j} + e_k + 1, n'_{k,j} y'_{k,j} + \tau(t_k))
\end{aligned} \tag{12}$$

The posterior marginal probability of $p'_{k,j}$ is

$$P''_{k,j} = \Pr(\beta_k = \beta_{k,j} | t_k) = \frac{p'_{k,j} u(t_k)^{\beta_{k,j}-1} \beta_{k,j}^{e_k} \frac{(n'_{k,j} y'_{k,j})^{n'_{k,j}+1}}{\Gamma(n'_{k,j}+1)} \frac{\Gamma(n''_{k,j}+1)}{(n''_{k,j} y''_{k,j})^{n''_{k,j}+1}}}{\sum_{j=1}^{S_k} p'_{k,j} u(t_k)^{\beta_{k,j}-1} \beta_{k,j}^{e_k} \frac{(n'_{k,j} y'_{k,j})^{n'_{k,j}+1}}{\Gamma(n'_{k,j}+1)} \frac{\Gamma(n''_{k,j}+1)}{(n''_{k,j} y''_{k,j})^{n''_{k,j}+1}}} \tag{13}$$

2.2. Component Posterior Predictive Distribution and Dynamic System Residual Life Distribution

In order to calculate the dynamic system RLD $R_{\text{sys}}^{(t_{\text{now}})}(t)$, we should obtain the probabilities $P(C_t^k = l_k)$ first. $m_k^{(t_{\text{now}})} = e_k^{(t_{\text{now}})} + c_k^{(t_{\text{now}})}$ is the number of type k components in the monitored system, and make all right-censored observations $t_{k,1}^+, \dots, t_{k,c_k}^+$ equal to t_{now}^+ , so the observation vector becomes $t_k^{(t_{\text{now}})} = (t_{k,1}, \dots, t_{k,e_k^{(t_{\text{now}})}}^+, t_{\text{now}}^+, \dots, t_{\text{now}}^+)$. For any $t > t_{\text{now}}$, we get the posterior predictive distribution:

$$\begin{aligned}
 P(C_t^k = l_k \mid n_{k,j}^{(0)}, y_{k,j}^{(0)}, p_{k,j}^{(0)}, t_k^{(t_{\text{now}})}) \\
 &= \binom{c_k^{(t_{\text{now}})}}{l_k} \int [P(T^k > t \mid T^k > t_{\text{now}}, \beta_{k,j}, \lambda_k)]^{l_k} [P(T^k \leq t \mid T^k > t_{\text{now}}, \beta_{k,j}, \lambda_k)]^{c_k^{(t_{\text{now}})} - l_k} \\
 &\quad \times f(\lambda_k, \beta_{k,j} \mid n_{k,j}^{(0)}, y_{k,j}^{(0)}, p_{k,j}^{(0)}, t_k^{(t_{\text{now}})}) d\lambda_k \\
 &= \binom{c_k^{(t_{\text{now}})}}{l_k} \sum_{j=1}^{s_k} \sum_{n=0}^{c_k^{(t_{\text{now}})} - l_k} \binom{c_k^{(t_{\text{now}})} - l_k}{n} (-1)^n p_{k,j}^{(0)} u(t_k^{(t_{\text{now}})})^{\beta_{k,j}-1} \beta_{k,j}^{e_k} \frac{(n_{k,j}^{(0)} y_{k,j}^{(0)})^{n_{k,j}^{(0)}+1}}{\Gamma(n_{k,j}^{(0)}+1)} \\
 &\quad \times \frac{\Gamma(n_{k,j}^{(t_{\text{now}})}+1)}{\{(l_k+n)[t^{\beta_{k,j}} - (t_{\text{now}})^{\beta_{k,j}}] + n_{k,j}^{(t_{\text{now}})} y_{k,j}^{(t_{\text{now}})}\}^{n_{k,j}^{(t_{\text{now}})}+1}} \\
 &\quad \times \frac{\sum_{j=1}^{s_k} p_{k,j}^{(0)} u(t_k)^{\beta_{k,j}-1} \beta_{k,j}^{e_k} \frac{(n_{k,j}^{(0)} y_{k,j}^{(0)})^{n_{k,j}^{(0)}+1}}{\Gamma(n_{k,j}^{(0)}+1)} \frac{\Gamma(n_{k,j}^{(t_{\text{now}})}+1)}{(n_{k,j}^{(t_{\text{now}})} y_{k,j}^{(t_{\text{now}})})^{n_{k,j}^{(t_{\text{now}})}+1}}
 \end{aligned} \tag{14}$$

Where $n_{k,j}^{(t_{\text{now}})} = n_{k,j}^{(0)} + e_k^{(t_{\text{now}})}$ and $n_{k,j}^{(t_{\text{now}})} y_{k,j}^{(t_{\text{now}})} = n_{k,j}^{(0)} y_{k,j}^{(0)} + \sum_{i=1}^{e_k^{(t_{\text{now}})}} (t_{k,i})^{\beta_{k,j}} + c_k^{(t_{\text{now}})} (t_{\text{now}})^{\beta_{k,j}}$.

If one or several components have failed by t_{now} , the survival signature will change with the system reliability block diagram. Therefore, the survival signature $\Phi(l_1, l_2, \dots, l_K)$ will become the current survival signature $\Phi^{(t_{\text{now}})}(l_1, l_2, \dots, l_K)$. Then, the dynamic system RLD $R_{\text{sys}}^{(t_{\text{now}})}(t)$ can be determined:

$$R_{\text{sys}}^{(t_{\text{now}})}(t) = \sum_{l_1=0}^{c_1^{(t_{\text{now}})}} \dots \sum_{l_K=0}^{c_K^{(t_{\text{now}})}} \Phi^{(t_{\text{now}})}(l_1, l_2, \dots, l_K) \prod_{k=1}^K P(C_t^k = l_k \mid n_{k,j}^{(0)}, y_{k,j}^{(0)}, p_{k,j}^{(0)}, t_k^{(t_{\text{now}})}) \tag{15}$$

3. A Novel Component Importance Measure

A novel importance measure is introduced in [19], which is expressed as the difference between the probability that the system functions if component i is working and the probability that the system functions if the i component is not working:

$$RII_i(t) = P(T_{\text{sys}} > t \mid T_i > t) - P(T_{\text{sys}} > t \mid T_i < t) \tag{16}$$

Where $P(T_{\text{sys}} > t \mid T_i > t)$ is the probability that the system functions if component i works, and $P(T_{\text{sys}} > t \mid T_i \leq t)$ is the probability that the system functions if the component i has failed.

This measure expresses the importance of a specific component during the system survival time. In Section 2, we obtained the dynamic system RLD. Now, we introduce the results of that into (16):

$$RII_i^{(t_{\text{now}})}(t) = P(T_{\text{sys}} > t \mid T_i^k > t, t > t_{\text{now}}) - P(T_{\text{sys}} > t \mid T_i^k < t, t > t_{\text{now}})$$

$$\begin{aligned}
&= \sum_{l_1=0}^{c_1^{(t_{now})}} \cdots \sum_{l_k=0}^{c_k^{(t_{now})}-1} \cdots \sum_{l_K=0}^{c_K^{(t_{now})}} [\tilde{\Phi}_1^{(t_{now})}(l_1, \dots, l_k, \dots, l_K) - \\
&\quad \tilde{\Phi}_0^{(t_{now})}(l_1, \dots, l_k, \dots, l_K)] \prod_{k=1}^K P(C_t^k = l_k | n_{k,j}^{(0)}, y_{k,j}^{(0)}, P_{k,j}^{(0)}, t_k^{(t_{now})})
\end{aligned} \quad (17)$$

Where $\tilde{\Phi}_1^{(t_{now})}(l_1, \dots, l_i, \dots, l_K)$ represents the survival signature that component i of type k works; $\tilde{\Phi}_0^{(t_{now})}(l_1, \dots, l_i, \dots, l_K)$ represents the survival signature that component i of type k has failed.

Of course, we can maintain multiple components at the same time. RII in (17) can be extended to the effect of two components: component i of type k_1 and component j of type k_2 , which is expressed as the difference between the probability that the system functions if components i and j are working together and the probability that the system functions if components i and j are not working:

$$\begin{aligned}
RII_{i^{k_1}, j^{k_2}}^{(t_{now})}(t) &= P(T_{sys} > t | T_{i^{k_1}} > t \text{ and } T_{j^{k_2}} > t, t > t_{now}) - P(T_{sys} > t | T_{i^{k_1}} < t \text{ and } T_{j^{k_2}} < t, t > t_{now}) \\
&= \sum_{l_1=0}^{c_1^{(t_{now})}} \cdots \sum_{l_{k_1}=0}^{c_{k_1}^{(t_{now})}-1} \cdots \sum_{l_{j^{k_2}}=0}^{c_{j^{k_2}}^{(t_{now})}-1} \cdots \sum_{l_K=0}^{c_K^{(t_{now})}} [\tilde{\Phi}_{i^{k_1}, j^{k_2}, 1}^{(t_{now})}(l_1, \dots, l_{i^{k_1}}, \dots, l_{j^{k_2}}, \dots, l_K) \\
&\quad - \tilde{\Phi}_{i^{k_1}, j^{k_2}, 0}^{(t_{now})}(l_1, \dots, l_{i^{k_1}}, \dots, l_{j^{k_2}}, \dots, l_K)] \times \prod_{k=1}^K P(C_t^k = l_k | n_{k,j}^{(0)}, y_{k,j}^{(0)}, P_{k,j}^{(0)}, t_k^{(t_{now})})
\end{aligned} \quad (18)$$

Where $\tilde{\Phi}_{i,j,1}^{(t_{now})}(l_1, \dots, l_i, \dots, l_j, \dots, l_K)$ represents the survival signature that component i and component j are working together and $\tilde{\Phi}_{i,j,0}^{(t_{now})}(l_1, \dots, l_i, \dots, l_j, \dots, l_K)$ represents the survival signature that component i and component j are not working.

4. An Application to the System Maintenance

4.1. Method Description

In the following section, we use the system with two cross-linked modules to make an experiment. The structural block diagram of the system is shown in Figure 1. Once the maintenance activity starts, the experiment ends. The system RLD is used to calculate the expected operational cycle cost rate.

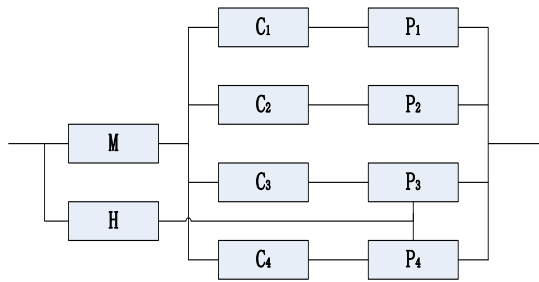


Figure 1. Reliability block diagram for a system with four component types M, H, C, and P

According to the prior parameters in Table 1, the failure times of the components in the system could be generated randomly.

While the maintenance strategy in [14] may not be practical in the actual optimal replacement, we still use that to verify the viability of the model in this article. [14] proposed a kind of condition-based maintenance policy, which uses the expected one-cycle cost rate as the unit cost rate to determine the optimal time to maintenance. Under $T_{sys}^{(t_{now})} = t_{sys}^{(t_{now})}$, the unit cost rate for planning maintenance at u time units after t_{now} is

$$g(u | T_{\text{sys}}^{(t_{\text{now}})} = t_{\text{sys}}^{(t_{\text{now}})}) = \begin{cases} c_p / (t_{\text{now}} + u), & \text{if } t_{\text{sys}}^{(t_{\text{now}})} \geq t_{\text{now}} + u \\ c_c / t_{\text{sys}}^{(t_{\text{now}})}, & \text{if } t_{\text{now}} < t_{\text{sys}}^{(t_{\text{now}})} < t_{\text{now}} + u \end{cases} \quad (19)$$

Table 1. Prior parameter sets for the four component types

k	$\beta_{k,j}$	$p_{k,j}^{(0)}$	$E[T^k]$	$n_{k,j}^{(0)}$	$y_{k,j}^{(0)}$	k	$\beta_{k,j}$	$p_{k,j}^{(0)}$	$E[T^k]$	$n_{k,j}^{(0)}$	$y_{k,j}^{(0)}$
M	2	0.2	6	2	45.84	C	1.5	0.2	8	1	26.38
	2.3	0.2		2	81.42		1.7	0.2		1	41.63
	2.5	0.2		2	118.92		2	0.2		1	81.49
	2.7	0.2		2	173.22		2.3	0.2		1	157.80
	3	0.2		2	303.34		2.5	0.2		1	244.11
H	1.1	0.2	10	1	13.09	P	1	0.2	4	1	4
	1.2	0.2		1	17.06		1.3	0.2		1	6.72
	1.3	0.2		1	22.13		1.5	0.2		1	9.33
	1.4	0.2		1	28.60		1.7	0.2		1	12.81
	1.5	0.2		1	36.87		2	0.2		1	20.37

The expected operational cycle cost rate is

$$g^{(t_{\text{now}})}(u) = E[g(u | T_{\text{sys}}^{(t_{\text{now}})})] = \frac{c_p}{t_{\text{now}} + u} R_{\text{sys}}^{(t_{\text{now}})}(t_{\text{now}} + u) + c_c \int_0^u \frac{1}{t_{\text{now}} + \theta} f_{\text{sys}}^{(t_{\text{now}})}(t_{\text{now}} + \theta) d\theta \quad (20)$$

We approximate the integral in (20) numerically by a sum, where $f_{\text{sys}}^{(t_{\text{now}})}(t)$ is calculated via differences of $R_{\text{sys}}^{(t_{\text{now}})}(t)$.

Here, we briefly describe the maintenance flow. Consider a grid of planned evaluation time points $\{m\delta : m \in \mathbb{N}_0\}$, in which δ is a sufficiently small time increment. In this paper, $\delta = 0.1$. During an operational cycle, $u_*^{(t_{\text{now}})} := \arg \min g^{(t_{\text{now}})}(u)$ should be recalculated at times $t_{\text{now}} = m\delta$. If $t_{\text{now}} < t_{k,i} < (m+1)\delta$, t_{now} is set to $t_{k,i}$. Next, we check if the system has failed or not. If the system has failed, a correction maintenance action is carried out. If the system has not failed, we calculate the optimal maintenance time $u_*^{(t_{\text{now}})}$ for $t_{\text{now}} = t_{k,i}$ and move back onto the grid afterwards, unless there is another component failure before the next planned evaluation time. Usually, $c_p \ll c_c$, and the cost parameters are chosen as $c_p = 1, c_c = 10$ in this paper.

In order to avoid the fact that the system could only choose the key components in cut sets for its preventive maintenance, we add a simple judgment condition in the maintenance flow above. If $m\delta = t_{\text{now}} < t_{3,C(P)} < (m+1)\delta$, the preventive maintenance starts at $t_{\text{now}} = m\delta$. $t_{3,C(P)}$ is the third component that will fail in type C or P , which can be determined by the failure times order of the components. Then, we can use the importance index to select the component or component groups that require preventive maintenance.

4.2. Method Verification

Now, the time to failure of the components of the system can be generated based on the prior information in Table 1, and the median prior expected failure times are $M = 3.7$, $H = 3.5$, $C = 6.4$, and $P = 2.1$. This is a failure history described in Figure 2, in which the system reliability functions and unit cost rate functions for $t_{\text{now}} = 0.1, 1, 2, 3, 4, 5, 6$ are displaced. In this operation cycle, we observe $C_3 = 6.56$, $P_2 = 5.52$, and $P_4 = 3.47$, and the failure times of other components are right-censored in system work until $t_{\text{now}} = 6.8$. Here, an observed failure for type C is close to the expected, while for type P , two observed failures are later than expected.

It is reflected in Figure 3(a) that large drops of $u_*^{(t_{\text{now}})}$ happen only at the failures of components P_4, P_2 , while a small drop of $u_*^{(t_{\text{now}})}$ happens at the failure of component C_3 . The failures are also reflected in Figure 2(a), where the reliability curve of $t_{\text{now}} = 3$ intersects with the following curves, and the curve of $t_{\text{now}} = 5$ crosses the curve of $t_{\text{now}} = 6$ as well. Since $u_*^{(t_{\text{now}})}$ does not change significantly at many times, $g_*^{(t_{\text{now}})}$ in Figure 3(b) steadily decreases with small drops corresponding

to the drops in $u_*^{(t_{now})}$. In time $t_{now} = 6.8$, $u_*^{(t_{now})}$ does not satisfy the condition of preventive maintenance, while $6.8 = m\delta = t_{now} < t_{3,P_1} < (m+1)\delta = 6.9$ satisfies the condition of preventive maintenance which we added. Once P_1 is a failure, the effective preventive maintenance can only be carried out for a cut set H and P_3 of the current system. Thus, at time $t_{now} = 6.8$, the preventive maintenance starts.

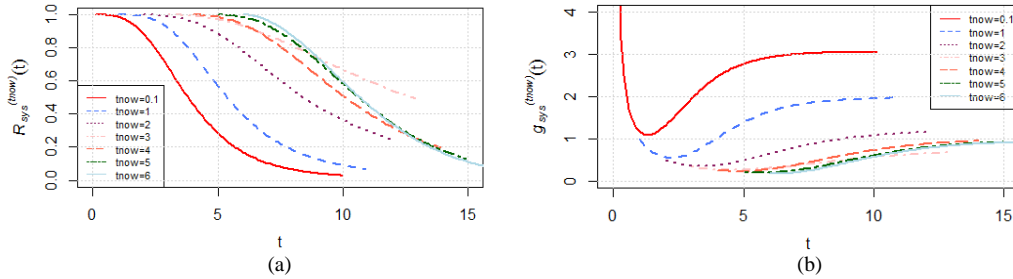


Figure 2. (a) System reliability functions; (b) System unit cost rate functions

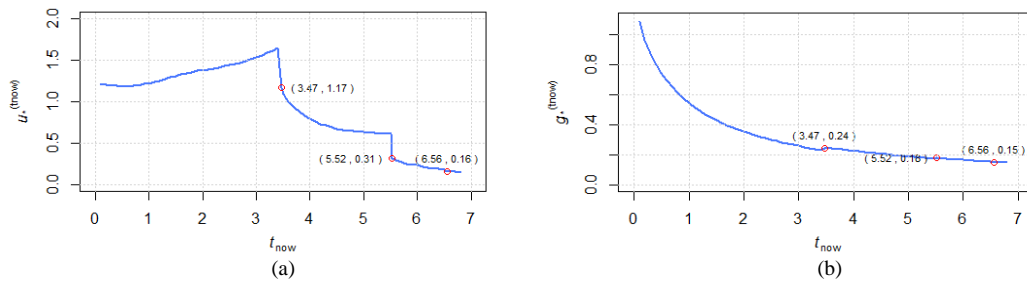


Figure 3. (a) The cost-optimal time to do maintenance; (b) The minimal expected operational cycle cost rate

Here, we explain that although the $u_*^{(t_{now})}$ derived from this article may not be the optimal value, it has already enabled us to make a judgment.

4.3. Component Importance Measure

From Figure 4(a), we obtain $RII_{P_3}^{(t_{now})}(t) > RII_{P_1}^{(t_{now})}(t) > RII_M^{(t_{now})}(t) > RII_{C_1}^{(t_{now})}(t) > RII_H^{(t_{now})}(t)$, and the maximum RII should be first performed on the maintenance order of components. Therefore, P_3 should be considered first when we select an individual component for preventive maintenance. Then, we can consider a group of components for preventive maintenance. In Figure 4(b), components M and H , M and P_3 , H and C_1 , H and P_1 , C_1 and P_3 , and P_1 and P_3 are cut sets in the current system, and we do not consider their maintenance. For component groups M and C_1 , M and P_1 , C_1 and P_1 , and H and P_3 , Figure 4(b) shows that the effects of components H and P_3 have the maximum impact on the system, so these two components should be considered first when we select a group of components for preventive maintenance.

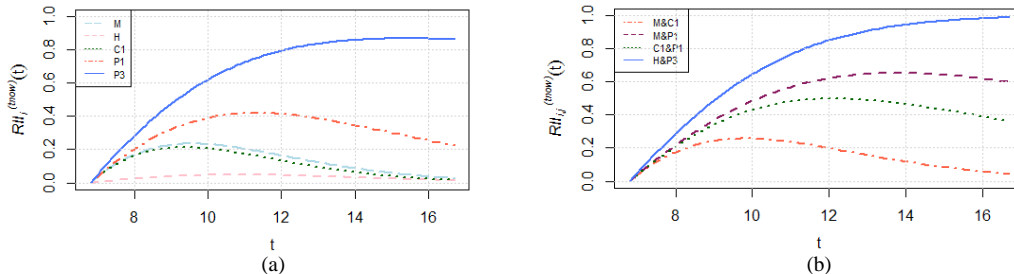


Figure 4. (a) Relative importance index of component M , H , C_1 , P_1 , and P_3 ; (b) Relative importance index of component groups (M, C_1) , (M, P_1) , (C_1, P_1) , and (M, P_3)

In the situation above, the failures are later than expected. In addition, the failures may be earlier than expected or as same as expected. Due to the analysis methods being similar, they are not recommended here.

5. Conclusions

A reliability assessment model is established for the system whose usual degradation signal is unavailable but whose component status can be monitored continuously. The failure time of each component types is modeled by a Weibull model with unknown shape and scale parameters. The system RLD is calculated based on the survival signature and Bayesian rules. Furthermore, the method can also be used to calculate component the importance by combining the system's reliability block diagram. The experiment results show that our model and estimation method are effective.

Future work can relax the assumption of precise component monitoring to consider the false positives and false negatives. In addition, the joint effect of component importance and maintenance cost can be considered.

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