

Determining Best Patch Management Software using Intuitionistic Fuzzy Sets with TOPSIS

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Abstract

Today's IT infrastructure demands for an automated yet stringently controlled solution to manage patches for vulnerable software applications. The use of patch management tools is the best practice that tests all the available patches before installation to ensure that the released patch will not break the existing software. However, the availability of several patch management software poses a challenge for the system administrator to decide which software facilitates the operational competence and effectiveness of the computer system in terms of revenue and system security. Therefore, selecting the appropriate patch management software that automatically patches all the Microsoft and non-Microsoft products simultaneously is an important and complex concern, leading to the multi-criteria decision approach. Here, we implement a hybrid approach that combines the intuitionistic fuzzy set and entropy weight-based multi-criteria decision making model with TOPSIS to select the best defense against vulnerabilities (or patch management software) in the group decision making environment. As most real world decision problems involve a group of decision makers that may have multiple opinions for individual criteria, the intuitionistic fuzzy weighted averaging operator is explicitly considered here and generates optimal weights for the attributes. A numerical example is provided to illustrate the application of the intuitionistic fuzzy TOPSIS method that helps identify the best patch management tool based on selected criteria.

Keywords: vulnerability; patch management; multi-criteria decision method; intuitionistic fuzzy; entropy; TOPSIS

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1. Introduction

Patch management (PM) has recently received considerable attention in both small- and large-scale enterprises. The PM is performed mainly by software companies as part of their internal efforts to solve problems with different software versions, to analyze current software programs, and to detect any possible deficiencies in security features or other upgrades. The way patches are distributed and deployed has experienced extreme changes over time. Conventionally, patches were provided on external media which could then be added to previously installed programs. Today, however, with the availability of Internet and cloud-hosting systems, everything has become discrete. Nowadays, there is no need for external media; rather, patches can be applied to software programs directly over the global IP network. This is the era of automated patch management software.

One of the major activities for PM success is technology or software that provides assistance in the completion of patch management tasks [1]. Technology notifies the availability of patches from a third-party vendor, deploys the patches onto vulnerable systems, maintains operational efficiency and effectiveness, overcomes security vulnerabilities, and maintains the stability of the production environment [2]. The most important aspect of technology is the selection of appropriate patch management software that balances the security and downtime risk of a security breach.

One of the most well-known studies [3] identified five important evaluation criteria that businesses should keep in mind when selecting the right patch management software. There are eight criteria or factors for selecting the secure patch

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management software [4].

In this paper, our major criteria on which scrutiny is performed are as follows: vulnerability prioritization, vulnerability analysis, and vendor participation. Recent studies suggest that almost 60% of organizations are not fully capable of identifying vulnerabilities that need updates. Thus, a patch management tool must be efficient in prioritizing vulnerabilities and patches according to the host environment. The second criterion addresses the need for risk assessment that determines the consequences of patch deployment (or change management) and develops hot fixes and walkthroughs automatically. The last criterion, i.e., vendor participation, encompasses all the third-party updates and patches relevant to the individuals' organization. All three criteria address the need for both IT security and IT operations.

The top five patch management software are taken here as alternatives, out of which the best tool that fulfils all three criteria is selected. The software are Solar Winds, Shavlik, LanDesk PM, Manage Engine Desktop Central, and GFI LanGuard. SolarWinds centralizes the entire patch process, from download to publish, and then to patch. It is integrated with Windows server update services and Microsoft update agent that automates patching on the basis of custom schedules [5]. Shavlik offers agentless patching such that patches are deployed to users' physical and virtual assets [6]. It performs well in large environments and maximizes users' effort by adding third-party patching to the system center configuration manager. Manage Engine integrates desktop and mobile devices to support Windows, Mac, and Linux as well as iOS and Android [7]. Built-in templates make it easy to manage patch installation for package creation. GFI LanGuard has the best user interface that supports many operating systems such as Mac OS X, Microsoft, Linux, and more than 60 third-party applications. It discovers risks and threats early to provide automate remediation for dynamic environments [8].

We propose a hybrid intuitionistic fuzzy set (IFS) entropy-based multi-criteria group decision making with the "Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)" method developed in [9] to select appropriate patch management software. Since it is a challenging issue to express the views of experts for criteria and the alternatives using crisp data, IFS is an effective way to implement multiple decision-making problems in an uncertain environment. We have considered the "intuitionistic fuzzy weighted averaging (IFWA) operator" to aggregate the opinions of all individuals such that the importance of each criteria and the impact of alternatives on criteria are ranked. On the other hand, TOPSIS provides both positive and negative ideal solutions for the multi-attribute decision making problem. Hence, it is assumed that the probability of success for patch management tool selection is high when TOPSIS is combined with IFS.

The paper is organized as follows: a detailed description of the proposed hybrid intuitionistic fuzzy with entropy and TOPSIS is presented in section 2. Section 3 demonstrates a case study of the proposed methodology, followed by the conclusion in Section 4.

2. Proposed Methodology

2.1. Preliminaries

2.1.1. Intuitionistic Fuzzy Set

Intuitionistic fuzzy set is an extension of the classical fuzzy set theory that was first introduced [10] to tackle fuzziness seen in the decision-making process. In a finite set P , an intuitionistic fuzzy set M is defined as:

$$M = \{(p, \mu_M(p), u_M(p)) | p \in P\}$$

Where $\mu_M(p)$ is the membership function and $u_M(p): P \rightarrow [0,1]$ is the non-membership function, with the condition that

$$0 \leq \mu_M(p) + u_M(p) \leq 1$$

For every IFS M in P , the third parameter $\pi_M(p)$ is known as the intuitionistic fuzzy index or hesitation degree of whether p belongs to M or not.

$$\pi_M(p) = 1 - \mu_M(p) - u_M(p) \quad (1)$$

It is obviously seen that for every $p \in P$, $0 \leq \pi_M(p) \leq 1$.

If $\pi_M(p)$ is small, information regarding p is more definite; meanwhile, this is not true when $\pi_M(p)$ becomes large. When $\mu_M(p) = 1 - u_M(p) \forall p$, the regular fuzzy set theory is improved [11]. Let M and N be the IFSs of the set P , and then multiplication operator is defined as follows [10]:

$$M \otimes N = \{ \mu_M(p) \cdot \mu_N(p), u_M(p) + u_N(p) \cdot \pi_M(p) \cdot \pi_N(p) | p \in P \} \quad (2)$$

2.1.2. Entropy of IFS

The entropy function [12] measures the discrete distribution uncertainty on the basis of classical statistical mechanics known for Boltzmann entropy. [13] proposed a Shannon's function based the non-probabilistic entropy formula of a fuzzy set on a finite universal set as $P = \{p_1, p_2, p_3, \dots, p_k\}$.

$$E_{LT}(M) = -i \sum_{x=1}^k \left[\mu_M(p_x) \ln \mu_M(p_x) + (1 - \mu_M(p_x)) \ln (1 - \mu_M(p_x)) \right], i > 0 \quad (3)$$

The work conducted in [13] was later extended in [14]. The authors provided multiple descriptions for entropy measure on IFSs(P). [15] measured the intuitionistic fuzzy entropy through the following equation, which satisfies the four axiomatic constraints:

$$E_{LT}^{IFS}(M) = -\frac{1}{k \ln 2} \sum_{x=1}^k \left[\mu_M(p_x) \ln \mu_M(p_x) + u_M(p_x) \ln u_M(p_x) - (1 - \pi_M(p_x)) \ln (1 - \pi_M(p_x)) - \pi_M(p_x) \ln 2 \right] \quad (4)$$

It is noted that $E_{LT}^{IFS}(M)$ is composed of the hesitancy degree and the fuzziness degree of the IFS M .

2.2. Proposed Intuitionistic Fuzzy and Entropy-based Decision Model with TOPSIS

Let $M = \{M_1, M_2, \dots, M_a\}$ denote the set of alternatives and $P = \{P_1, P_2, \dots, P_k\}$ denote the set of criteria. Here, we explain the step by step procedure for the intuitionistic fuzzy and entropy-based decision model with the TOPSIS method as shown in Figure 1.

Stage 1. Determine the weights of decision makers

If we have 'X' decision makers, then their importance is determined through linguistic terms expressed in intuitionistic fuzzy numbers.

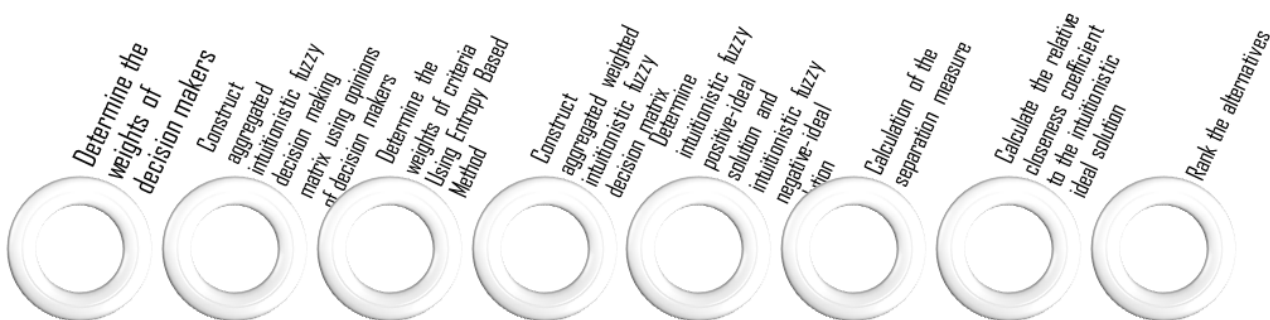


Figure 1. Steps for implementation of proposed model

If $D_i = (\mu_i, u_i, \pi_i)$ is an intuitionistic fuzzy number that ranks the i^{th} decision maker, then the weight of i^{th} decision maker can be obtained as:

$$\phi_i = \frac{\left(\mu_i + \pi_i \left(\frac{\mu_i}{\mu_i + u_i} \right) \right)}{\sum_{i=1}^l \left(\mu_i + \pi_i \left(\frac{\mu_i}{\mu_i + u_i} \right) \right)}, \quad \sum_{i=1}^l \phi_i = 1 \quad (5)$$

Stage 2. Construct an aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers

Let $U^{(i)} = (u_{xy}^{(i)})_{a \times k}$ be an intuitionistic fuzzy decision matrix of each decision maker. $\phi = \{\phi_1, \phi_2, \phi_3, \dots, \phi_l\}$ is the weight of each decision maker, and $\sum_{i=1}^l \phi_i = 1$, $\phi_i \in [0, 1]$. In the process of group decision-making, all the individual opinions are consolidated into a group opinion such that an aggregated intuitionistic fuzzy decision matrix is created via the IFWA operator proposed in [16].

$$U = (u_{xy})_{a \times k}$$

Where

$$\begin{aligned} u_{xy} &= IFWA_{\phi}(u_{xy}^1, u_{xy}^2, \dots, u_{xy}^l) \\ &= \phi_1 u_{xy}^1 \oplus \phi_2 u_{xy}^2 \oplus \dots \oplus \phi_l u_{xy}^l \\ &= \left[1 - \prod_{i=1}^l (1 - \mu_{xy}^{(i)})^{\phi_i}, \prod_{i=1}^l (\mu_{xy}^{(i)})^{\phi_i}, \prod_{i=1}^l (1 - \pi_{xy}^{(i)})^{\phi_i} - \prod_{i=1}^l (\pi_{xy}^{(i)})^{\phi_i} \right] \end{aligned} \quad (6)$$

Here, $u_{xy} = (\mu_{M_x}(p_y), \pi_{M_x}(p_y))$, $(x = 1, 2, \dots, a; y = 1, 2, \dots, k)$.

The aggregated intuitionistic fuzzy decision matrix can be defined as follows:

$$U = \begin{bmatrix} (\mu_{M_1}(p_1), u_{M_1}(p_1), \pi_{M_1}(p_1)) & (\mu_{M_1}(p_2), u_{M_1}(p_2), \pi_{M_1}(p_2)) & \dots & (\mu_{M_1}(p_n), u_{M_1}(p_n), \pi_{M_1}(p_n)) \\ (\mu_{M_2}(p_1), u_{M_2}(p_1), \pi_{M_2}(p_1)) & (\mu_{M_2}(p_2), u_{M_2}(p_2), \pi_{M_2}(p_2)) & \dots & (\mu_{M_2}(p_n), u_{M_2}(p_n), \pi_{M_2}(p_n)) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{M_a}(p_1), u_{M_a}(p_1), \pi_{M_a}(p_1)) & (\mu_{M_a}(p_2), u_{M_a}(p_2), \pi_{M_a}(p_2)) & \dots & (\mu_{M_a}(p_n), u_{M_a}(p_n), \pi_{M_a}(p_n)) \end{bmatrix} \quad (7)$$

$$U = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1k} \\ u_{21} & u_{22} & \dots & u_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ u_{a1} & u_{a2} & \dots & u_{ak} \end{bmatrix}$$

Stage 3. Determine the criteria weights using the entropy-based method

Let $q_y^i = \{\mu_y^{(i)}, u_y^{(i)}, \pi_y^{(i)}\}$ be an intuitionistic fuzzy number assigned to criterion P_j by the i^{th} decision maker. Then, the weights of the criteria are calculated by using the well-known entropy method [17].

$$E_{LT}^{IFS}(C_y) = -\frac{1}{a \ln 2} \sum_{x=1}^a \left[\mu_{xy}(C_x) \ln \mu_{xy}(C_x) + u_{xy}(C_x) \ln u_{xy}(C_x) - (1 - \pi_{xy}(C_x)) \ln (1 - \pi_{xy}(C_x)) - \pi_{xy}(C_x) \ln 2 \right] \quad (8)$$

Where $y = 1, 2, \dots, k$ and $1/(a \ln 2)$ is a constant, which assures $0 \leq E_{LT}^{IFS}(C_y) \leq 1$.

Therefore, the degree of divergence (d_y) of the average intrinsic information provided by the corresponding performance ratings on criterion C_y can be defined as:

$$(d_y) = 1 - E_{LT}^{IFS}(C_y), \quad y = 1, 2, \dots, k \quad (9)$$

The value of (d_y) represents the inherent contrast intensity of criterion C_y , and then the entropy weight of the y^{th} criterion is:

$$w_y = \frac{d_y}{\sum_{y=1}^k d_y} \quad (10)$$

Stage 4. Construct an aggregated weighted intuitionistic fuzzy decision matrix

After the weights of criteria (W), the aggregated weighted intuitionistic fuzzy decision matrix is constructed based on the following definition:

$$\begin{aligned} \bar{E} &= W^T \otimes \bar{D} = W^T \otimes [\bar{p}_{xy}]_{a \times k} = [\bar{p}_{xy}] \\ W &= (w_1, w_2, \dots, w_y, \dots, w_k) \\ \bar{p}_{xy} &= \langle \bar{\mu}_{xy}, \bar{u}_{xy}, \bar{\pi}_{xy} \rangle = \left\langle 1 - (1 - \mu_{xy})^{w_x}, u_{xy}^{w_x}, 1 - \left(1 - (1 - \mu_{xy})^{w_x} - u_{xy}^{w_x}\right) \right\rangle, \quad w_y > 0 \end{aligned} \quad (11)$$

Stage 5. Determine the intuitionistic fuzzy positive ideal solution (IFPIS) and intuitionistic fuzzy negative ideal solution (IFNIS)

In general, the evaluation criteria Y_1 and Y_2 are the benefit criteria and cost criteria, respectively. A^+ is an intuitionistic fuzzy positive ideal solution, and A^- is an intuitionistic fuzzy negative ideal solution. The IFS theory and standard of the conventional TOPSIS method defines IFPIS and IFNIS as:

$$\begin{aligned} A^+ &= (\bar{\mu}_{A^+W}(C_y), \bar{u}_{A^+W}(C_y)) \text{ and } A^- = (\bar{\mu}_{A^-W}(C_y), \bar{u}_{A^-W}(C_y)) \\ \bar{\mu}_{A^+W}(C_y) &= ((\max \bar{\mu}_{A^+W}(C_y) | y \in Y_1), (\min \bar{\mu}_{A^+W}(C_y) | y \in Y_2)) \\ \bar{u}_{A^+W}(C_y) &= ((\max \bar{u}_{A^+W}(C_y) | y \in Y_1), (\min \bar{u}_{A^+W}(C_y) | y \in Y_2)) \\ \bar{\mu}_{A^-W}(C_y) &= ((\max \bar{\mu}_{A^-W}(C_y) | y \in Y_1), (\min \bar{\mu}_{A^-W}(C_y) | y \in Y_2)) \\ \bar{u}_{A^-W}(C_y) &= ((\max \bar{u}_{A^-W}(C_y) | y \in Y_1), (\min \bar{u}_{A^-W}(C_y) | y \in Y_2)) \end{aligned} \quad (12)$$

Stage 6. Calculate the distance measures

The distance measures given in [18] are used to measure the separation between alternatives. Once the distance measure is chosen, the separation measures are calculated as $d_{IFS}(M_i, A^+)$, $d_{IFS}(M_i, A^-)$ for each alternative on the basis of intuitionistic fuzzy positive and negative-ideal solutions. Here, the intuitionistic Euclidean distance (2000) is used to rank all alternatives.

$$d_{IFS}(M_i, A^+) = \sqrt{\sum_{y=1}^k \left[(\mu_{M_iW}(C_y) - \mu_{A^+W}(C_y))^2 + (u_{M_iW}(C_y) - u_{A^+W}(C_y))^2 + (\pi_{M_iW}(C_y) - \pi_{A^+W}(C_y))^2 \right]} \quad (13)$$

$$d_{IFS}(M_i, A^-) = \sqrt{\sum_{y=1}^k \left[\left(\mu_{M_i, W}(C_y) - \mu_{A^-, W}(C_y) \right)^2 + \left(u_{M_i, W}(C_y) - u_{A^-, W}(C_y) \right)^2 + \left(\pi_{M_i, W}(C_y) - \pi_{A^-, W}(C_y) \right)^2 \right]}$$

Stage 7. Calculate the relative closeness coefficient of each alternative

The relative closeness coefficient of an alternative M_i with respect to the intuitionistic fuzzy positive ideal solution A^+ is defined as follows:

$$CC_{i^+} = \frac{d_{IFS}(M_i, A^-)}{d_{IFS}(M_i, A^+) + d_{IFS}(M_i, A^-)}, \quad 0 \leq CC_{i^+} \leq 1, \quad i = 1, 2, 3, \dots, a \quad (14)$$

A larger value of CC_{i^+} indicates that an alternative is closer to IFPIS and farther away from IFNIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of CC values. The most preferred alternative is the one with the highest CC value.

Stage 8. Rank the alternatives

After evaluating the relative closeness coefficient of each alternative, alternatives are ranked according to descending order of CC_{i^+} values.

3. Numerical Illustration

IT organizations aim to select the most appropriate patch management tool for analyzing patches that needs to be rolled out. After pre-evaluation, five patch management software are shortlisted as alternatives. To rank the tools, a team of five decision-makers or experts is created. The three most important criteria derived from rigorous literature review are:

- P_1 : Vulnerability prioritization (determining which updates will be rolled out to which devices)
- P_2 : Vulnerability analysis (identifying relevant vulnerabilities and updates)
- P_3 : Vendor participation (covering a wide variety of vendor software)

The selection of the patch management tool follows the given steps:

Step 1 Determine the weights of the decision makers

Firstly, the degree of importance for each decision maker is calculated using Equation (5). Table 1 presents the linguistic terms used for determining the significance of each decision maker. Table 2 and Table 3 shows the linguistic terms used by each decision maker for weighing the criteria and alternatives respectively.

$$\lambda_{DM_1} = \frac{0.9}{0.9 + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right) + 0.9 + \left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45} \right) \right) + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right)} = 0.23$$

$$\lambda_{DM_2} = \frac{\left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right)}{0.9 + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right) + 0.9 + \left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45} \right) \right) + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right)} = 0.202$$

$$\lambda_{DM_3} = \frac{0.9}{0.9 + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right) + 0.9 + \left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45} \right) \right) + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2} \right) \right)} = 0.23$$

$$\lambda_{DM_4} = \frac{\left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45}\right)\right)}{0.9 + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2}\right)\right) + 0.9 + \left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45}\right)\right) + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2}\right)\right)} = 0.135$$

$$\lambda_{DM_5} = \frac{\left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2}\right)\right)}{0.9 + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2}\right)\right) + 0.9 + \left(0.5 + 0.05 \times \left(\frac{0.5}{0.5 + 0.45}\right)\right) + \left(0.75 + 0.05 \times \left(\frac{0.75}{0.75 + 0.2}\right)\right)} = 0.202$$

Table 1. Importance of decision makers and their weights

	1	2	3	4	5
Linguistic terms	Very significant	Significant	Very significant	Medium	Significant
	0.23	0.2	0.23	0.13	0.2

Table 2. Linguistic terms for decision makers and criteria

Linguistic terms	IFNs
Very significant	(0.9, 0.1)
Significant	(0.75, 0.2)
Medium	(0.5, 0.45)
Insignificant	(0.35, 0.6)
Very insignificant	(0.1, 0.9)

Table 3. Linguistic terms for alternatives

Linguistic terms	IFNs
Extremely good	(1, 0)
Very very good	(0.9, 0.1)
Very good	(0.8, 0.1)
Good	(0.7, 0.2)
Medium good	(0.6, 0.3)
Fair	(0.5, 0.4)
Medium bad	(0.4, 0.5)
Bad	(0.25, 0.6)
Very bad	(0.1, 0.75)
Very very bad	(0.1, 0.9)

Step 2 Construct the aggregated intuitionistic fuzzy decision matrix based on the opinions of decision makers

The ratings provided by the decision makers (as shown in Table 4) are combined to create the intuitionistic fuzzy decision matrix through Equation (7) as shown in Table 5.

Table 4. Ratings of the alternatives

Criteria	Patch management solutions	Decision makers				
		DM ₁	DM ₂	DM ₃	DM ₄	DM ₅
P ₁	SolarWinds	G	VG	G	EG	VVG
	LANDesk	MG	G	F	VG	G
	Shavlik	VVG	VG	VG	VVG	G
	ManageEngine Desktop Central	MG	G	G	MG	F
	GFI LanGuard	F	MG	MG	F	MB
P ₂	SolarWinds	MG	G	MG	G	VG
	LANDesk	F	MG	G	F	G
	Shavlik	VG	G	VG	G	VG
	ManageEngine Desktop Central	F	F	MG	G	MG
	GFI LanGuard	MB	F	F	VB	B
P ₃	SolarWinds	VG	G	VG	VVG	VG
	LANDesk	G	MG	MG	EG	MG
	Shavlik	VG	VG	G	EG	VVG
	ManageEngine Desktop Central	VG	G	G	G	VVG
	GFI LanGuard	G	G	MG	G	F

Table 5. Weights assigned to each alternative

	P_1	P_2	P_3
SolarWinds	(1, 0, 0)	(0.68, 0.213, 0.106)	(0.799, 0.118, 0.084)
LANDesk	(0.684, 0.239, 0.077)	(0.613, 0.283, 0.104)	(1, 0, 0)
Shavlik	(0.828, 0.118, 0.054)	(0.768, 0.129, 0.104)	(1, 0, 0)
ManageEngine Desktop Central	(0.627, 0.27, 0.103)	(0.572, 0.326, 0.102)	(0.778, 0.151, 0.071)
GFI LanGuard	(0.563, 0.383, 0.054)	(0.385, 0.5, 0.115)	(0.641, 0.256, 0.103)

Step 3 Weights of criteria using entropy

Weights computed using Equations (8)-(10) are shown in Table 6.

Table 6. Weights of the criteria

Criteria	P_1	P_2	P_3
Entropy ($E_{LT}^{IFS}(C_j)$)	0.7957	0.8793	0.7285
Distance measure (d_j)	0.2043	0.1207	0.2715
Weights (w_j)	0.3425	0.2024	0.4551

Step 4 Construct the aggregated weighted intuitionistic fuzzy decision matrix

Equation (11) is used to aggregate the weighted intuitionistic fuzzy decision matrix, as shown in Table 7.

Table 7. Aggregated weighted IFS-entropy decision matrix

Stage 4		P_1			P_2			P_3		
		μ	u	π	μ	u	π	μ	u	π
Aggregated weighted IFS-Entropy Decision Matrix	SolarWinds	0.2054	0.7669	0.0277	0.0719	0.9014	0.0267	0.5188	0.3906	0.0906
	LANDesk	0.1145	0.8513	0.0342	0.0632	0.9117	0.0251	0.6993	0.2763	0.0244
	Shavlik	0.1525	0.814	0.0335	0.0858	0.8846	0.0296	0.6993	0.2763	0.0244
	Manage Engine Desktop Central	0.1053	0.8586	0.0361	0.0591	0.9213	0.0196	0.5093	0.4218	0.0689
	GFI LanGuard	0.0881	0.8886	0.0233	0.0381	0.945	0.0169	0.4007	0.5096	0.0897

Step 5 Obtain intuitionistic fuzzy positive ideal solution and intuitionistic fuzzy negative ideal solution

Here, criteria are classified into benefit criteria and cost criteria; thus, we calculate the positive and negative ideal solutions using Equation (12), as shown in Table 8.

Table 8. IFPIS & IFNIS

Stage 5	P_1			P_2			P_3		
	μ	u	π	μ	u	π	μ	u	π
A^+	0.201139	0.147262	0.147507	0.259515	0.393396	0.201139	0.147262	0.147507	0.259515
A^-	0.059838	0.151868	0.163756	0.022305	0	0.059838	0.151868	0.163756	0.022305

Step 6 Calculate the separation measures and relative closeness coefficient to the intuitionistic ideal solution

Finally, the intuitionistic Euclidean distance is used to calculate the separation measures and closeness coefficients for each alternative through Equations (13) and (14), and the results are shown in Table 9.

Table 9. Separation measures and closeness coefficients

Separation Measures & Closeness Coefficient		SolarWinds	LANDesk	Shavlik	Manage Engine Desktop Control	GFI LanGuard
	$d_{IFS}(A_i, A^+)$	0.4485	0.2446	0.3529	0.4485	0.2446
	$d_{IFS}(A_i, A^-)$	0.3837	0.3897	0.5039	0.3837	0.3897
	CC_{i^+}	0.3841	0.4047	0.5131	0.3841	0.4047

Step 7 Alternatives are ranked

We determine the relative closeness coefficients and then rank the five alternatives in descending order of CC_{i^+} values. The alternatives were ranked as $A_3 > A_2 = A_5 > A_4 = A_1$ or Shavlik > LANDesk = GFI LanGuard > ManageEngine Desktop Central = SolarWinds. We can conclude that Shavlik is the most suitable patch management tool among the alternatives.

4. Conclusions

Many applications of intuitionistic fuzzy sets have been proposed in recent years as a multi-criteria decision-making approach in the broad areas of engineering and management. In this paper, we have evaluated and ranked the most appropriate patch management software with the help of the proposed hybrid intuitionistic entropy-based decision making model and TOPSIS. In information theory, entropy has been used as a measurement in relation to the average information available for the source. Intuitionistic fuzzy set theory first values the decision makers and evaluates the importance of each option on the basis of pre-determined criteria, and then it uses Shannon's entropy to obtain the optimal criteria weights. We have aggregated the weighted intuitionistic fuzzy and entropy-based matrix via the IFWA operator. Finally, TOPSIS methodology is used to determine the best alternative. Here, Shavlik is observed to be the most superior patch management software among others and has the capability to fulfill the mentioned criteria.

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