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Reliability Analysis based on Inverse Gauss Degradation Process and Evidence Theory

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Abstract

The degradation analysis of products has been demonstrated as a significant toolkit for reliability analysis. Data from the same batch of products in different working environments cannot be directly used to analyze product reliability. In this paper, motivated by this circumstance, we first assume that degradation data sets from different working environments are subject to different inverse Gaussian process models, and maximum likelihood estimation is used to obtain multiple model parameters. Secondly, we construct evidence by quantifying different information of products, apply the evidence theory to fuse model parameters, and then analyze the reliability of products from the same batch. Finally, we use performance degradation data of the laser to illustrate the method.

Keywords: degenerate modeling; inverse Gaussian process; evidence theory; data fusion; reliability analysis

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Nomenclature

IG	Inverse Gaussian
MLE	Maximum likelihood estimation
BPA	Basic probability assignment
ER	Evidential reasoning
PDF	Probability density function
MLE	Maximum likelihood estimation
DF	Probability density function
$Y(t)$	Degradation process
$\mathrm{IG}(\Delta\Lambda, \lambda\Delta\Lambda^2)$	Inverse Gaussian process
λ	Scale parameter of the Inverse Gaussian process
$\Lambda_C(t)$	Linear mean function of the Inverse Gaussian process
$\Lambda_M(t)$	Monotonic mean function of the Inverse Gaussian process
$r(t)$	Degradation rate
$L(\theta_{IG})$	Likelihood function of degradation model parameter θ_{IG}
θ_{IG}	Vector of model parameters
Δy	Degradation increment of a degradation process $Y(t)$
$f\left(y(t)\right)$	Probability density function of $y(t)$
$\Phi(\bullet)$	Cumulative distribution of a standard normal distribution
Θ	Frame of discernment
$R(t)$	Reliability function under the degradation process

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$m_1(A_i)$	Basic probability assignment function defined by data sample size evidence of A_i
$m_2(A_i)$	Basic probability assignment function defined by expertise evidence A_i
$m_3(A_i)$	Basic probability assignment function defined by expertise evidence of A_i

1. Introduction

Performance degradation is the evolution process of the state during the failure process. It is different from traditional failure-based product life information data. Performance degradation data mainly concerns the failure process information related to product life. Through the analysis and study of the failure mechanism of the product, the related performance characteristics that can reflect the life or reliability of the product are selected. The characteristic variables are called performance degradation, and the failure process of the product is described quantitatively. Compared with the traditional product information based on failure life, performance-based degradation products include more product implicit information, making up for the effect of failure data to ignore the initial performance difference between products. With the fast development of the defense industry and advanced manufacturing technology, the failure data of products is difficult to collect.

With the development of monitoring technology, more and more performance degradation information can be collected in the working state of the product. Based on performance degradation data, modeling and analysis product reliability has become significant. Performance degradation is a highly correlated physical variable with product life. We usually describe the variation laws of the degradation path of products with time as a stochastic process. In order to reduce the number of parameters of the Wiener degradation process while considering the relationship between individual variance and population variance, life data and degradation data were fused to analyze the reliability of the blowout preventer valve [1]. Unlike the Wiener degradation process, the gamma degradation process describes a monotonic degradation process that makes up for the discontinuity of the Wiener process [2]. However, engineering practice shows that some degenerate data cannot be simply described by the Wiener or gamma process. The IG process also has independent incremental properties. Some scholars have introduced it into the modeling of product performance degradation. The IG process is an independent but not necessarily identically distributed gamma process in the extreme case [3]. It can be used to analyze remaining useful life and product reliability [4-7]. The stochastic degradation process model was used to analyze the reliability of the product, and the key problem was to obtain more accurate model parameters with the data collected. The concept of degenerated mean function was proposed, three different degradation rates were discussed, and the IG degradation process model parameters were estimated by the Bayesian theory [8]. The IG process was used to model the degradation data of oil and gas pipelines corrosion, and the empirical maximization algorithm and particle filter algorithm were combined to estimate the parameters of the model [9]. A competitive failure model was constructed with the IG degradation process, and MLE was used to evaluate the model parameters [5]. Based on the inverse Gaussian process, a two-stage Bayesian method was introduced to implement parameter estimation [10]. It is noteworthy that all the analysis methods listed above have been studied under the same means of data collection, from the same or similar work environment. In order to analyze the product status under different environments and fuse multi-source information, the evidence theory was proposed and applied to reliability data processing. Evidence theory is a complete theory dealing with uncertainty. It can emphasize not only the objectivity of things, but also the subjectivity of human's estimation of things [11-13]. The prior data required by the evidence theory is more intuitive and easier to obtain than that in the probabilistic inference theory, and the description of uncertain problems is flexible. By synthesizing the knowledge and data of different experts or data sources, the description of uncertain problems is more flexible and convenient. The concept of attribute weight was proposed, and ER was used to fuse the model parameters [14]. The angle cosine similarity coefficient and its similarity matrix were used as the weight of the data to improve the D-S evidence theory and analyze the reliability of a diesel engine [15].

We have noticed little research on discrepant data in different working environments. If the discrepant data has been forcibly used to estimate model parameters, the results of the analysis will be inaccurate. In practical engineering, especially for complex products or systems, reliability analyses under different working environments will contain various kinds of uncertainty, and common analysis methods are not perfect for multi-source information processing in different environments. The evidence theory has certain advantages in dealing with probabilistic uncertainty, and it can effectively fuse multi-source uncertain information in different environments. However, it is still a challenge to evaluate the reliability of performance degradation products with discrepant data under different environments.

The outline of this paper is as follows. In Section 2, the IG degradation process is introduced into data modeling, and MLE is adopted to estimate the model parameters. In Section 3, the model parameters are fused with the combined evidence theory and the information of different products in the same batch. In Section 4, the idea of the algorithm is summarized. In Section 5, the method is examined by a numerical example involving a laser. In Section 6, conclusions for performance degradation product reliability analysis based on evidence theory are presented.

2. Inverse Gauss Process and Evidence Theory Degenerate Modeling Method

The following will be divided into two parts to discuss. The first part establishes the IG process model and estimation parameter, and the other part elaborates on the model parameter fusion algorithm based on the evidence theory.

2.1. The Model of the IG Process

For a batch of performance degradation products, only a small amount of degradation data can be obtained due to technical and budgetary constraints. Suppose the degradation data set of the product is $\{(t_i, y_i) i = 1, 2, \cdots, n\}$ $t_i \in R,$ and the i^{th} monitoring time $y_i \in R,$ is the product degradation at the corresponding time. The product degradation path $\{Y(t), t \geq 0\}$ obeys the inverse Gaussian process degradation model and has the following properties [16]:

- $Y(t) \equiv 0$.
- For any $t_2 > t_1 \geq s_2 > s_1$, $Y(t_2) - Y(t_1)$ and $Y(s_2) - Y(s_1)$ are independent degenerate increments.
- The degenerate increments $Y(t_2) - Y(t_1) \sim IG(\Lambda(t), \lambda\Lambda(t)^2)$, $\Lambda(t) = \Lambda(t_2) - \Lambda(t_1)$.

Parametric $\Lambda(t)$ is a function of monotonous growth that has a clear physical meaning and describes the mean of the degenerate process. We can use $\Lambda(t)$ to define the different product degradation rate $r(t)$. When the degradation rate $r(t)$ is the linear and the monotonic function separately, the corresponding degradation mean functions are obtained [8]:

$$\Lambda_C(t) = \int r_c(t)\mathrm{d}t = \mu t, \ \mu > 0 \tag{1}$$

$$\Lambda_M(t) = \int r_M(t)\mathrm{d}t = \left(\frac{t}{\eta}\right)^{\beta}, \ \beta > 0, \ \eta > 0 \tag{2}$$

Where β and η are the shape parameter and scale parameter respectively, and μ is a constant. We have the probability density function of $y(t)$:

$$f(y(t)|\Lambda(t), \lambda) = \sqrt{\frac{\lambda\Lambda(t)^2}{2\pi y(t)^3}} \exp\left\{-\frac{\lambda\left(y(t) - \Lambda(t)\right)^2}{2y(t)}\right\} \tag{3}$$

When the product performance degradation reaches the preset failure threshold, $T = \{t | y(t) \geq D\}$, and we define this as product failure. Considering the monotony of the IG process, the reliability function of the product is defined as [3]:

$$\begin{aligned} R_{IG}(t) &= P(T_{IG} \geq t) = P(y(t) \leq D) \\ &= \int_0^D \sqrt{\frac{\lambda\Lambda(t)^2}{2\pi y(t)^3}} \exp\left\{-\frac{\lambda\left(y(t) - \Lambda(t)\right)^2}{2y(t)}\right\} \mathrm{d}y \\ &= \phi\left[\sqrt{\frac{\lambda}{D}}\left(D - \Lambda(t)\right)\right] + e^{(2\pi\Lambda(t))} \cdot \phi\left[-\sqrt{\frac{\lambda}{D}}\left(D + \Lambda(t)\right)\right] \end{aligned} \tag{4}$$

2.2. Model Parameter Estimation

We have product degradation data $\{y(t_1), y(t_2), y(t_3), \cdots, y(t_n)\}$, and the products have an independent degenerate increment $\Delta Y = \{\Delta y_1, \Delta y_2, \cdots, \Delta y_n\}$.

$y(0) \equiv 0$, and $\Delta y = y(t+\Delta t) - y(t) \sim IG(\Delta\Lambda, \lambda\Delta\Lambda^2)$ obeys an IG distribution.

We use MLE to calculate IG degenerate model parameters. When the product degenerate increment $\Delta y_i \sim IG(\Delta\Lambda, \lambda\Delta\Lambda^2),\ i=1,\cdots,n,\ \Delta\Lambda > 0,\ \lambda > 0$, obtain the likelihood function of parameters $\theta_{IG} = (\Lambda(t), \lambda)$:

$$L(\theta_{IG}) = \prod_{i=1}^{n} \sqrt{\frac{\lambda\Lambda(\Delta t_i)^2}{2\pi\Lambda(\Delta t_i)^3}} \exp\left\{ -\frac{\lambda\left(\Delta y_i(t) - \Lambda(\Delta t_i)\right)^2}{2\Delta y_i(\Delta t_i)} \right\} \tag{5}$$

Let $L(\theta_{IG})$ have a partial derivative of 0 on $\Delta\Lambda(t),\ \lambda$, and then the MLE of the IG model parameter $L(\theta_{IG})$ is:

$$\Lambda\Delta(t) = \frac{1}{n}\sum_{i=1}^{n} \Delta y_i(t) \tag{6}$$

$$\lambda = \left(\left(\frac{1}{n}\sum_{i=1}^{n} \Delta y_i(t) \right)^2 \left(\frac{1}{n}\sum_{i=1}^{n} \frac{1}{\Delta y_i(t)} \right) - \frac{1}{n}\sum_{i=1}^{n} \Delta y_i(t) \right)^{-1} \tag{7}$$

If the different product degradation data does not have the discrepancy or the discrepant data can be ignored, the data can be used directly to estimate the parameters $\Lambda\Delta(t)$ and λ.

For the discrepant data of the same batch of products, different degenerate data should have different degrees of importance in the estimation of the parameter for the degeneration model.

3. Multi-Source Information Fusion based on Evidence Theory

Evidence theory has a certain advantage in dealing with multiple source uncertain information: it satisfies a weaker condition than the Bayesian probability theory. It can emphasize the objectivity of things and the subjectivity of human's estimation of things.

The set of possible even information in different environments is defined as the frame of discernment. It is also the assumed space for information that all even may imply, denoted as Θ. A subset of the frame of discernment is called a proposition. Evidence is designed by experts based on propositions. Define the basic probability assignment to describe the trust degree based on the frame of discernment, represented as a probability that possible events given in a function, usually recorded as the m function, reflect the reliability of A. The following conditions must be satisfied [17]:

$$m: 2^{\Theta} \to (0,1),\ m(\varnothing) = 0,\ \sum\nolimits_{X \subseteq \Theta} m(X) = 1 \tag{8}$$

2^{Θ} is a power set of the frame of discernment Θ, and a power set is a collection of subsets of a set.

The discrepant degenerate data are subject to different IG degradation process models, and the multiple degenerate data has multiple model parameters. Let each model parameter be an element of the frame of discernment $\Theta = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$. BPA describes the trust degree to discrepant data under the different evidence. Determine the probability of the estimated parameters in each working environment for different evidence. Obtain the model parameters for products in different working environments, the volume of the sample data, and the parameter deviation degree. The rules of definition are as follows.

3.1. Data Sample Capacity

Under the frame of discernment, the BPA function of the sample size of the product data in different environments can be obtained by calculating the percentage of the sample size directly. However, this can easily cause small sample information to be diluted and even ignored in the fusion. We calculate the logarithm of the sample size and the corresponding percentage and then obtain the BPA function under the evidence of product data sample capacity [16].

Suppose there are n groups of sample degenerate data, $Y=\{y_1,y_2,\cdots,y_n\}$, $y_i=\{\partial_{i1},\partial_{i2},\cdots,\partial_{ij}\}$, $(i=1,2\cdots,n)$, i is the number of sample degenerate data in different environment experimental products, j is the number of discrete points in the degraded monitoring time, and ∂_{ij} is the ith product degradation in the monitoring time j. For the parameters estimated in the degenerate data of group i, the BPA function under the evidence of data sample size is as follows:

$$m_1(A_i)=\frac{\ln|y_i|}{\sum_{i=1}^{n}\ln|y_i|},\ \forall A_i\subseteq\Theta \tag{9}$$

3.2. Parameter Deviation Degree

To avoid biased estimates of the parameters and large deviations of the product path and overall degeneracy path, the corresponding data should be assigned a small weight.

For multiple model parameters $\alpha_1,\alpha_2,\cdots,\alpha_n$, we get the mean value $\alpha=\frac{1}{n}\sum_{i=1}^{n}\alpha_i$.

Calculate the deviation of each parameter from the mean value; the greater the deviation from the total mean value, the higher the uncertainty and the lower the level of importance. Therefore, according to the reciprocal of distance as a criterion, define the BPA function under the evidence of the parameter deviation degree as

$$m_2(A_i)=\frac{\frac{1}{\sqrt{(\alpha_i-\alpha)^2}}}{\sum_{i=1}^{n}\frac{1}{\sqrt{(\alpha_i-\alpha)^2}}},\ \forall A_i\subseteq\Theta \tag{10}$$

3.3. Expertise

According to the authority of experts, the importance of each product in the same batch of products is given. The closer the data work environment is to the current environment, the higher the weight that should be given. The BPA function defined by expertise evidence is

$$m_3(A_i)=P(A_i),\ \forall A_i\subseteq\Theta \tag{11}$$

Where $P(\bullet)$ is a probability function.

3.4. Dempster's Combination Rule

In solving practical problems, experts design various evidence, calculate every evidence of power set elements of trust, trust all the evidence, and then calculate the comprehensive trust of all evidence of events.

For $A_i\subseteq\Theta$ under the frame of discernment, there is a finite number of BPA $m_1,m_2,\cdots,m_n$, and Dempster's combination rule is shown below:

$$(m_1\oplus m_2\oplus\cdots\oplus m_n)(A_i)=\frac{1}{K}\sum_{A_1\cap A_2\cap\cdots\cap A_n}m_1(A_1)\bullet\cdots\bullet m_n(A_n) \tag{12}$$

Where $K=\sum_{A_1\cap A_2\cap\cdots\cap A_n\neq\varnothing}m_1(A_1)\bullet\cdots\bullet m_n(A_n)=1-\sum_{A_1\cap A_2\cap\cdots\cap A_n=\varnothing}m_1(A_1)\bullet\cdots\bullet m_n(A_n)$, called the normalization factor.

4. General Idea of Algorithm

- Highline cable has ideal flexibility, which means it only withstands axial tension and cannot undergo compression or anti-bending [10].

- Based on the product data, an IG degradation model is established, and model parameters are estimated for each set of product data.
- Based on Dempster's combination rule and BPA, the reasonable weight values of the model parameters are obtained.
- Under the given product failure threshold D and the fused parameters, the reliability of the same batch of products is analyzed.

The general idea of the algorithm is shown in Figure 1.

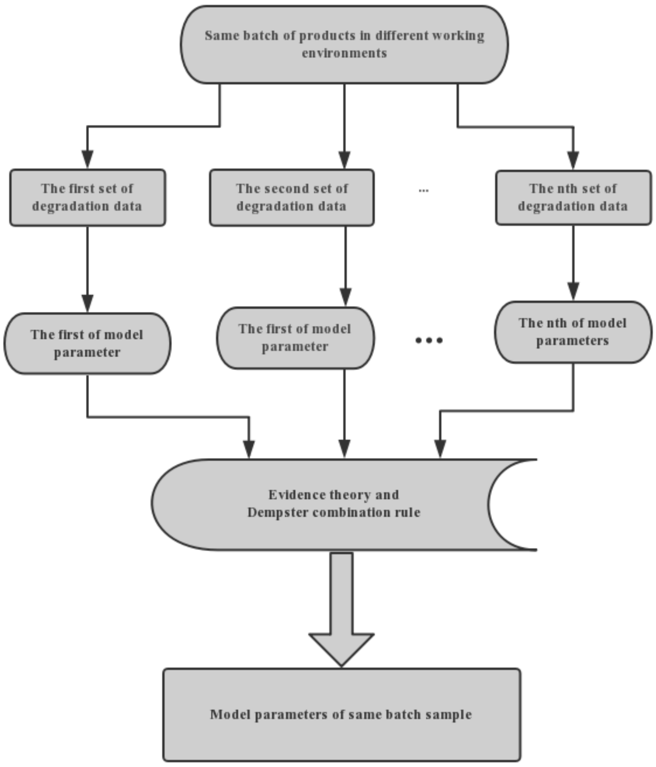

Figure 1. Algorithm flow chart

5. Case Analyses

The brightness of the laser will gradually decrease with an increase in the use time. To keep the laser brightness at a constant working level, we need to increase the working current over time. Define product failure when the operating current increases to a fixed threshold. The laser data used in this paper was obtained in different environments with different sample sizes. We use this data as an example to prove the validity and feasibility of the proposed method.

A performance indicator increases the percentage of the operating current. The degradation data of four test products are measured in this paper. In addition to one product, the other three products were monitored at six equal interval dispersion points. The following Table 1 and Figure 2 show the degradation test data and the corresponding degradation path.

An IG process model is used to fit the degenerated data of the laser, and it is shown that the degradation process conforms to the IG process [2].

According to expert experience, the process of laser degradation obeys a linear path. Therefore, establishing an IG degradation process mode $IG(\Lambda(t), \lambda\Lambda(t)^2)$, $\Lambda(t) = \mu t$, $u > 0$ is a constant degradation rate. Using the MLE method to obtain the parameter $\Theta_{IG} = (\Lambda(t), \lambda)$ in the model, obtain four sets of parameter values, shown in Table 2.

Table 1. Performance degradation data of lasers

	Time (h)					
	500	1000	1500	2000	2500	3000
Sample1	1.07	1.77	2.40	3.02	3.75	4.76
Sample2	0.61	1.77	2.58	3.88	4.63	5.62
Sample3	0.93	1.96	3.29	4.11	4.91	5.84
Sample4	1	1.96	2.84	4.01		

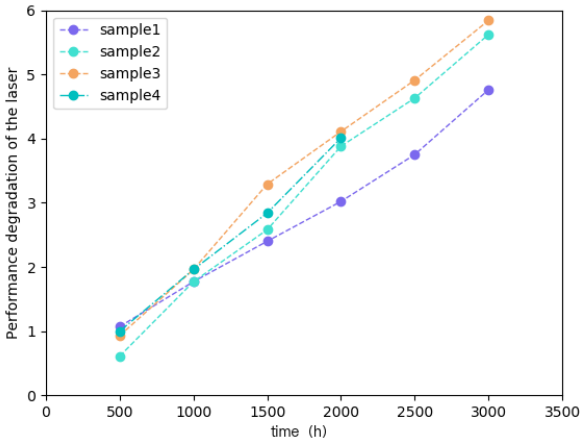

Figure 2. Performance degradation trajectory of laser

Table 2. MLE of model parameters

Parameters	Sample1	Sample2	Sample3	Sample4
$\Lambda(t)=\mu t$	0.790 t	0.930 t	0.973 t	1.002 t
λ	0.051	0.050	0.028	0.011

Under the framework of evidence reasoning, the method proposed in this paper is used. Based on the product degradation test data and the BPA of different evidences, the sample size, deviation information of parameter mean value, and expertise are calculated. Dempster's combination rule is applied to get the proportion of importance of each product model parameter after the fusion of multi-source evidence, the fused parameter $\mathsf{L}(t)$ and λ are shown in Table 3 and Table 4 respectively.

Table 3. Parameter $\Lambda(t)$ Dempster's combination

	Evidence			
	Sample size	Mean deviation	Expert experience	After fusion
Sample1 parameter	0.265	0.04	0.2	0.02
Sample2 parameter	0.265	0.8	0.4	0.87
Sampe3 Parameter	0.265	0.11	0.3	0.09
Sample4 parameter	0.205	0.05	0.1	0.02

Table 4. Parameter λ Dempster's combination

	Evidence			
	Sample size	Mean deviation	Expert experience	After fusion
Sample1 parameter	0.265	0.20	0.2	0.15
Sample2 parameter	0.265	0.21	0.4	0.31
Sample3 parameter	0.265	0.46	0.3	0.50
Sample4 parameter	0.205	0.13	0.1	0.04

The method of weighted averages is utilized to obtain the parameters of the IG process model as $\Lambda(t)=0.9325t$, $\lambda=0.0376$.

According to engineering experience, the operating current of the laser is increased by 10% from the initial current. We judge the failure of the product. Substituting the failure threshold $(D=10)$ and the corresponding model parameters $\Lambda(t)=0.9325t$, $\lambda=0.0376$ into (4), the analytical expression of the laser reliability under multi-source information is obtained:

$$R_{IG}(t)==\phi\left[\frac{10-0.932t}{16.31}\right]+e^{(\pi\cdot 1.86t)}\cdot\phi\left[-\frac{(10-0.932t)}{16.31}\right] \tag{13}$$

In order to further prove the validity and practicality of the proposed method, the same four sets of laser degradation data are used, but the sample data of these products are directly combined into one sample set. The parameters of the corresponding model are obtained as respectively. Use OpenBUGS software and Monte Carlo simulations to obtain the model parameters under the above two methods. Assume that the parameters are unconditionally a priori and each group produces 5000 degenerate data, and then calculate the average degradation value of the product. Finally, compare the results with a set of product performance degradation data collected in real-time, simulation comparison results and corresponding trajectories are shown in Table 5 and Figure 3.

It can be seen that the infused degradation samples obtained using the simulation are affected by the first set of deviation samples, and this causes the macro forecast value to be relatively low. Through expert experience, we know that sample3 is closest to the new sample working environment. Therefore, sample3 should receive more attention. After assigning the weight, the fusion sample obtained from the simulation is closest to the new real sample. To some extent, the impact of discrepancies in product sample data is avoided.

Table 5. Fusion and infusion information degradation prediction comparison

	Time (h)					
	500	1000	1500	2000	2500	3000
New data	0.74	1.85	3	3.8	4.3	5.6
Infusion evidence	0.8	1.82	2.68	3.4	4.0	4.8
Fusion evidence	0.7	1.81	2.8	3.7	4.4	5.7

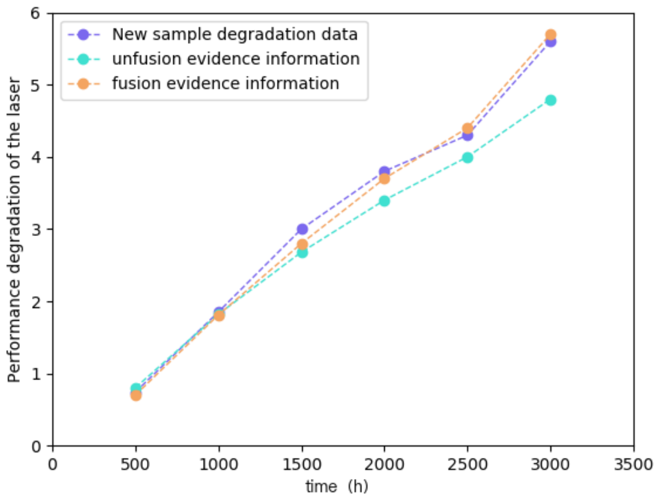

Figure 3. Fusion and infusion information degradation prediction comparison

6. Conclusions

To address discrepancies in degradation data of the same batch of products, a degenerate modeling method combining evidence theory and IG process is proposed. After the fusion of data, the degradation model can be established more accurately, making up for the problem of inaccurate product reliability analysis caused by the simple combination of data. By displaying and analyzing the degenerate data of a laser, fusion of different information from the same batch product data to achieve accurate analysis product reliability index provides a novel idea.

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