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A Preventive Maintenance Model Subject to Sequential Inspection for a Three-Stage Failure Process

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Abstract

For a system subject to gradual degradation, it can be in one of three different states: normal, minor defective, or severe defective stages, which can overall be referred to as a three-stage failure process. For this system, periodic inspection may not be the most ideal policy. Fewer inspections will lead to lower costs if the system is in the normal stage, but if the system is in the defective stage, frequent inspections are recommended to prevent failure. Therefore, it may be more economical to take the sequential time $T_j\,(j=0,1,2,\cdots)$ as the inspection interval. In view of this, a preventive maintenance (PM) model subject to sequential inspection for a three-stage failure process is proposed. Two-level sequential inspections, postponed maintenance and opportunistic maintenance (OM), are introduced into the PM model. The minor inspection is taken at the successive time $S_j\,(j=0,1,2,\cdots)$, where $S_j=\sum_{k=0}^{j}T_k$, $T_0=0$, and $S_0=0$. Minor inspection is an imperfect inspection that can identify the minor defective stage with a certain probability but can reveal other two stages completely. The nth major inspection is taken to substitute for the $A_n{}^{\text{th}}\,(n=0,1,2,\cdots)$ minor inspection, where $A_n=\sum_{k=0}^{n}N_k$, $N_0=0$, and $A_0=0$. Major inspection is a perfect inspection that can distinguish the state of the system perfectly. Once the severe defective stage is identified, the inspection is stopped and the maintenance action is postponed to the next OM if the time to the next OM is less than a threshold level; otherwise, the system is maintained immediately. A numerical example is given to demonstrate the proposed model by comparing with other models and analysing the influence of the parameters on the expected cost.

Keywords: preventive maintenance; three-stage failure process; sequential inspection; postponed maintenance; opportunistic maintenance

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1. Introduction

Preventive maintenance (PM) is widely adopted in actual industrial practice to improve reliability and economy. As an important part of PM, inspection has received extensive attention. An effective inspection policy could provide efficient information about the system state for maintenance decisions. According to the system state information, an appropriate maintenance action needs to be taken to prevent an unexpected failure as well as avoid excessive maintenance. Considering this situation, a PM model for the three-stage failure process is developed. Two-level sequential inspections, postponed maintenance and OM, are introduced into this model to reduce the long run expected cost per unit time.

Many publications established PM models that focus on the binary state (normal or failed) system [1-2]. However, the degradation process of a system is usually divided into more than two stages in industrial maintenance practice [3-4]. For example, what has been observed in practice is that the states of the system are usually represented by three colors (green, yellow, and red) [5]. Furthermore, there are some n redundant subsystems of a dependable system in nuclear power plants. When some m of the n subsystems fail, instead of failing immediately, the whole system is then degraded to operate with lower (n-m) redundancies, which results in reduced reliability. This situation can be described as the delay time stage. The

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delay time conception was first defined and used to predict inspection frequency by Christer [6-7]. Wang summarized the recent advances in delay-time-based maintenance modeling and extended the two-stage failure process into a three-stage failure process [5, 8]. Many researchers have developed numerous PM models based on delay time conception [9-13].

Because it is easy to implement, periodic inspection is commonly adopted in industrial applications, and it has been widely discussed in literature [14-17]. However, periodic inspection may not be the most ideal policy for a three-stage failure process. In the early operation period, the system in the normal stage is a high probability event, and a longer inspection interval may save costs. After a period of running, shortening the inspection interval along with increasing the runtime could prevent failure. It is therefore rational to consider that taking the sequential time T_j as the inspection interval may be more economical. This inspection policy is named sequential inspection [2]. It is thought that sequential inspection is more appropriate for a system whose aging property has to be estimated with the information from inspection [18]. Optimization of the sequential inspection scheme for a system subject to gradual degradation is studied [19-20]. Sequential inspection is considered in this PM model for a three-stage failure process.

In industrial practice, there is usually a planned down time for a large system after running for a certain time. For instance, a nuclear power plant will shut down for refueling, which is referred to as nuclear power plant refueling outage [21]. This planned down time creates an opportunity to arrange a comprehensive maintenance. Because this maintenance does not require extra downtime, its cost can be significantly reduced. This kind of maintenance is called opportunistic maintenance (OM). Considering the economy of the OM, a maintenance action can be allowed to be postponed to the next OM if the time to the next OM is less than a predetermined threshold. The advantages of postponed maintenance are summarized as follows: avoiding production disruption or unnecessary or ineffective replacement, preparing for replacement, extending component life, and waiting for an opportunity [22].

The remaining part of this paper is organized as follows. Modeling assumptions are given in Section 2. Section 3 studies the PM model subject to two-level sequential inspections for a three-stage failure process. For the purpose of comparison, some other models are introduced in Section 4. Section 5 presents a numerical example to demonstrate the proposed model. Section 6 concludes the paper and discusses further research key points.

Table 1. Nomenclature

X_m	Random variables representing the durations of the m^{th} stage of the system $(m=1,2,3)$
$f_{X_m}(x)$	Probability density function (pdf) of X_m
T_k	Interval from (k-1)$^{\text{th}}$ minor inspection to k^{th} minor inspection
N_n	There are N_n minor inspection intervals between (n-1)$^{\text{th}}$ major inspection and n^{th} major inspection, $N_0=0$
T	Minor inspection intervals sequence $T=\{T_1,T_2,\cdots,T_k,\cdots\}$
N	Major inspection intervals sequence $N=\{N_1,N_2,\cdots,N_n,\cdots\}$
S_j	Minor inspection is implemented at successive time S_j , $S_j=\sum_{k=0}^{j}T_k$, $T_0=0$, $S_0=0$
A_n	The n^{th} major inspection happens at the time when $A_n{}^{\text{th}}$ minor inspection should be taken, $A_n=\sum_{k=0}^{n}N_k$
T_{OM}	The OM interval
τ	Random time to the next OM
t	The threshold deciding whether to wait for OM
α	Probability of minor inspection detecting minor defective stage
T_{cr}	Random time when the corrective replacement happens for the failed system
T_{ir}	Random time when the inspection replacement is taken for the defective system
T_{or}	Random time when the opportunistic replacement is implemented for the defective system
c_{mi}	Cost of a minor inspection
c_{ma}	Cost of a major inspection
c_{cr}	Cost of a corrective replacement
c_{ir}	Cost of an inspection replacement
c_{or}	Cost of an opportunistic replacement

2. Model Assumptions

This section provides some modeling assumptions. Most of the assumptions are consistent with what has been observed in the practical industry and have been adopted in previous research [2, 11].

- For a system subject to a three-stage failure process, it can be in one of three stages: normal, minor defective, or severe defective stages. These three stages are independent.
- Two-level sequential inspections are adopted in this model. An imperfect inspection is implemented at the successive time $S_j\,(j=0,1,2,\cdots)$, which is named the minor inspection. When the A_n^{th} $(n=0,1,2,\cdots)$ minor inspection is to be conducted, instead of taking the minor inspection, a major inspection is performed. Major inspection is a perfect inspection, and its cost is higher than the cost of minor inspection. Both inspections can be performed online or the cost due to downtime can be regarded as a part of the inspection cost; therefore, the downtime does not appear on the timeline.
- The minor inspection can identify the minor defective stage with a known probability α, while the major inspection can reveal the minor defective stage completely. Both the normal and severe defective stages can be identified by either inspection perfectly.
- If the minor defective stage is identified by inspection, the next inspection intervals may be adjusted according to the information derived from inspection, such as by shortening inspection intervals. Once the severe defective stage is identified, stop the inspection and postpone replacement to the next OM if the time to the next OM is less than a threshold t; otherwise, the system is replaced immediately.
- The failure of a system can be identified immediately once it happens. The only maintenance action is to replace the system with a new one in this paper. Replacing the failed system is called corrective replacement. When the severe defective stage is found by inspection, immediate replacement is referred to as inspection replacement, while replacement that is postponed to the next OM is opportunistic replacement. The cost of corrective replacement is significantly higher than that of inspection replacement, while the opportunistic replacement cost is lower than the inspection replacement cost.

3. A PM Model Subject to Sequential Inspection for a Three-Stage Failure Process

In this section, the long run expected cost per unit time is chosen as a measure to optimize decision variables. First, the probabilities of the system being replaced under different scenarios are derived. Then, the expected replacement cycle cost and the expected replacement cycle length of the system can be obtained. Lastly, the relationship among the long run expected cost per unit time and decision variables is established.

3.1. Probability of Corrective Replacement

When the system fails, a corrective replacement is taken. Depending on whether the minor or severe defective stage is identified by inspection, different corrective replacements are discussed.

(1) Corrective replacement happens in (S_{A_n+i}, S_{A_n+i+1}). As shown in Figure 1(a), the normal stage ends in (S_{A_n+j}, S_{A_n+j+1}). Then, the minor defective stage continues until the severe defective stage starts in (S_{A_n+i}, S_{A_n+i+1}). Subsequently, the system fails and corrective replacement is implemented. In this process, neither the minor nor severe defective stage is detected by inspection.

$$P(S_{A_n+i} < T_{cr} < S_{A_n+i+1}) = \sum_{j=0}^{i} P\left\{\begin{array}{l} S_{A_n+j} < X_1 < S_{A_n+j+1} \\ i-j \text{ minor inspections didn't identify} \\ \text{the minor defective stage} \\ S_{A_n+i} - X_1 < X_2 < S_{A_n+i+1} - X_1 \\ X_3 < S_{A_n+i+1} - X_1 - X_2 \end{array}\right\} \tag{1}$$

$$= \sum_{j=0}^{i} \int_{S_{A_n+j}}^{S_{A_n+j+1}} \int_{\Delta(S_{A_n+i}-x)}^{S_{A_n+i+1}-x} \int_{0}^{S_{A_n+i+1}-x-y} (1-\alpha)^{i-j} f_{X_1}(x) f_{X_2}(y) f_{X_3}(z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

Where $j=0,1,\cdots,i$; $n=0,1,\cdots$; $i=0,1,\cdots,N_{n+1}-1$; and $N_{n+1}=A_{n+1}-A_n$. To ensure that the lower limit of integration is no less than 0, $\Delta(x)$ is defined:

$$\Delta(x)=\begin{cases}0, & x<0\\ x, & x\geq 0\end{cases} \tag{2}$$

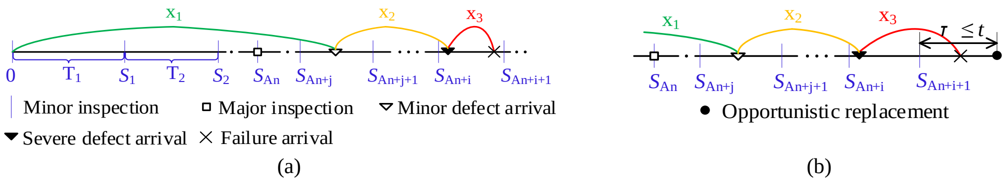


Figure 1. (a) Corrective replacement happens in (S_{A_n+i},S_{A_n+i+1}); (b) Corrective replacement is taken in $(S_{A_n+i+1},S_{A_n+i+1}+\tau)$

(2) Corrective replacement is taken in $(S_{A_n+i+1},S_{A_n+i+1}+\tau)$. The severe defective stage is identified by inspection at S_{A_n+i+1}, but the replacement is arranged at the next OM because the time to the next OM is less than the threshold t. However, the system fails before the arrival of the next OM.

$$P(S_{A_n+i+1}<T_{cr}<S_{A_n+i+1}+\tau)=\sum_{j=0}^{i}P\left\{\begin{array}{l} S_{A_n+j}<X_1<S_{A_n+j+1}\\ i-j \text{ minor inspections didn't identify}\\ \text{the minor defective stage}\\ S_{A_n+i}-X_1<X_2<S_{A_n+i+1}-X_1\\ S_{A_n+i+1}-X_1-X_2<X_3<S_{A_n+i+1}+\tau-X_1-X_2\\ \tau\leq t\end{array}\right\}$$

$$=\sum_{j=0}^{i}\int_{S_{A_n+j}}^{S_{A_n+j+1}}\int_{\Delta(S_{A_n+i}-x)}^{S_{A_n+i+1}-x}\int_{S_{A_n+i+1}-x-y}^{S_{A_n+i+1}+\tau-x-y}\int_0^t(1-\alpha)^{i-j}f_{X_1}(x)f_{X_2}(y)f_{X_3}(z)f(\tau)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau \tag{3}$$

Where $f(\tau)$ denotes the pdf of τ.

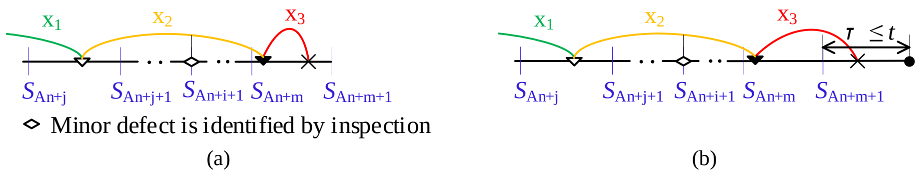


Figure 2. (a) Corrective replacement is implemented in (S_{A_n+m},S_{A_n+m+1}); (b) Corrective replacement is performed in $(S_{A_n+m+1},S_{A_n+m+1}+\tau)$

(3) Corrective replacement is implemented in (S_{A_n+m},S_{A_n+m+1}). Before the failure arrival, the minor defective stage is identified by inspection at S_{A_n+n+1}. Then, shorter inspection intervals may be adopted. However, the system fails directly after entering the severe defective stage as shown in Figure 2(a).

$$P(S_{A_n+m}<T_{cr}<S_{A_n+m+1})=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}P\left\{\begin{array}{l} S_{A_n+j}<X_1<S_{A_n+j+1}\\ i-j \text{ minor inspections didn't}\\ \text{identify the minor defective stage}\\ S_{A_n+m}-X_1<X_2<S_{A_n+m+1}-X_1\\ X_3<S_{A_n+m+1}-X_1-X_2\end{array}\right\}$$

$$=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}\int_{S_{A_n+j}}^{S_{A_n+j+1}}\int_{S_{A_n+m}-x}^{S_{A_n+m+1}-x}\int_0^{S_{A_n+m+1}-x-y}\delta(i,j,N_{n+1})f_{X_1}(x)f_{X_2}(y)f_{X_3}(z)\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{4}$$

Where $m = i+1, i+2, \cdots$, and $\delta(i, j, N_{n+1})$ is defined as:

$$\delta(i, j, N_{n+1}) = \begin{cases} \alpha(1-\alpha)^{i-j}, & i < N_{n+1}-1 \\ (1-\alpha)^{i-j}, & i = N_{n+1}-1 \end{cases} \tag{5}$$

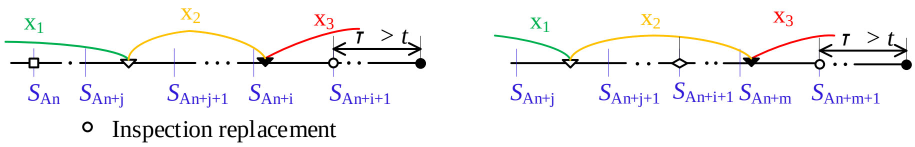
Where $i = N_{n+1}-1$ denotes the inspection implemented at S_{A_n+i+1} is a major inspection, which can identify the minor defective stage perfectly. While $i < N_{n+1}-1$, the minor inspection performed at S_{A_n+i+1} can distinguish the minor defective stage with a certain probability α .

(4) Corrective replacement is performed in $(S_{A_n+m+1}, S_{A_n+m+1}+\tau)$. Both the minor and severe defective stages are identified by inspections. It is decided to postpone replacement until the next OM since $\tau < t$. As Figure 2(b) shows, in this case, the system fails while waiting for the next OM.

$$P(S_{A_n+m+1} < T_{cr} < S_{A_n+m+1}+\tau) = \sum_{i=0}^{N_{n+1}\text{-}1}\sum_{j=0}^{i} P\left\{\begin{array}{l} S_{A_n+j} < X_1 < S_{A_n+j+1} \\ i-j \quad \text{minor} \quad \text{inspections} \quad \text{didn't} \\ \text{identify} \quad \text{the} \quad \text{minor} \quad \text{defective} \quad \text{stage} \\ S_{A_n+m} - X_1 < X_2 < S_{A_n+m+1} - X_1 \\ S_{A_n+m+1} - X_1 - X_2 < X_3 < S_{A_n+m+1} + \tau - X_1 - X_2 \\ \tau \le t \end{array}\right\}$$
$$= \sum_{i=0}^{N_{n+1}\text{-}1}\sum_{j=0}^{i} \int_{S_{A_n+j}}^{S_{A_n+j+1}} \int_{S_{A_n+m}-x}^{S_{A_n+m+1}-x} \int_{S_{A_n+m+1}-x-y}^{S_{A_n+m+1}+\tau-x-y} \int_0^t \delta(i, j, N_{n+1}) f_{X_1}(x) f_{X_2}(y) f_{X_3}(z) f(\tau) \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau \tag{6}$$

3.2. Probability of Inspection Replacement

When the severe defective stage is identified by inspection, inspection replacement is implemented immediately if the time to the next OM is greater than the threshold *t*. Depending on whether the minor defective stage is identified, two situations are discussed.



(a) (b)

Figure 3. (a) Inspection replacement happens at S_{A_n+i+1} ; (b) Inspection replacement is taken at S_{A_n+m+1}

(1) Inspection replacement happens at S_{A_n+i+1}, as shown in Figure 3(a), and the minor defective stage is not detected by either inspection before implementing the inspection replacement.

$$P(T_{ir} = S_{A_n+i+1}) = \sum_{j=0}^{i} P\left\{\begin{array}{l} S_{A_n+j} < X_1 < S_{A_n+j+1} \\ i-j \quad \text{minor} \quad \text{inspections} \quad \text{didn't} \quad \text{identify} \\ \text{the} \quad \text{minor} \quad \text{defective} \quad \text{stage} \\ S_{A_n+i} - X_1 < X_2 < S_{A_n+i+1} - X_1 \\ X_3 > S_{A_n+i+1} - X_1 - X_2, \quad \tau > t \end{array}\right\}$$
$$= \sum_{j=0}^{i} \int_{S_{A_n+j}}^{S_{A_n+j+1}} \int_{\Delta(S_{A_n+i}-x)}^{S_{A_n+i+1}-x} \int_{S_{A_n+i+1}-x-y}^{\infty} \int_t^{T_{OM}} (1-\alpha)^{i-j} f_{X_1}(x) f_{X_2}(y) f_{X_3}(z) f(\tau) \mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau \tag{7}$$

(2) Inspection replacement is taken at S_{A_n+m+1}. Before the system enters the severe defective stage, the minor defective

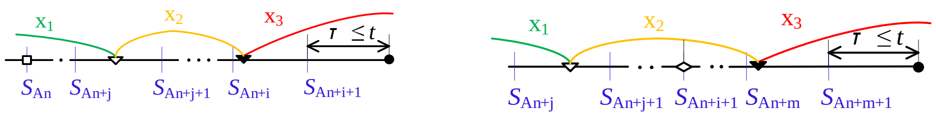
stage is identified in Figure 3(b).

$$P(T_{ir}=S_{A_n+m+1})=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}P\left\{\begin{array}{l} S_{A_n+j}<X_1<S_{A_n+j+1} \\ i-j \quad \text{minor} \quad \text{inspections} \quad \text{didn't} \\ \text{identify} \quad \text{the} \quad \text{minor} \quad \text{defective} \quad \text{stage} \\ S_{A_n+m}-X_1<X_2<S_{A_n+m+1}-X_1 \\ X_3>S_{A_n+m+1}-X_1-X_2, \quad \tau>t \end{array}\right\} \tag{8}$$

$$=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}\int_{S_{A_n+j}}^{S_{A_n+j+1}}\int_{S_{A_n+m}-x}^{S_{A_n+m+1}-x}\int_{S_{A_n+m+1}-x-y}^{\infty}\int_{t}^{T_{OM}}\delta(i,j,N_{n+1})f_{X_1}(x)f_{X_2}(y)f_{X_3}(z)f(\tau)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau$$

3.3. Probability of Opportunistic Replacement

If the severe defective stage is identified and the time to the next OM is less than the threshold t, instead of replacing the system immediately, the inspection is stopped and replacement is postponed to the next OM. Then, the opportunistic replacement is implemented successfully before the arrival of failure.



(a) (b)

Figure 4. (a) Opportunistic replacement is Implemented at $S_{A_n+i+1}+\tau$; (b) Opportunistic replacement is performed at $S_{A_n+m+1}+\tau$

(1) Opportunistic replacement is implemented at $S_{A_n+i+1}+\tau$, as Figure 4 (a) shows, and only the severe defective stage is identified by inspection.

$$P(T_{or}=S_{A_n+i+1}+\tau)=\sum_{j=0}^{i}p\left\{\begin{array}{l} S_{A_n+j}<X_1<S_{A_n+j+1} \\ i-j \quad \text{minor} \quad \text{inspections} \quad \text{didn't} \quad \text{identify} \\ \text{the} \quad \text{minor} \quad \text{defective} \quad \text{stage} \\ S_{A_n+i}-X_1<X_2<S_{A_n+i+1}-X_1 \\ X_3>S_{A_n+i+1}+\tau-X_1-X_2, \quad \tau\le t \end{array}\right\} \tag{9}$$

$$=\sum_{j=0}^{i}\int_{S_{A_n+j}}^{S_{A_n+j+1}}\int_{\Delta(S_{A_n+i}-x)}^{S_{A_n+i+1}-x}\int_{T_{A_n+i+1}+\tau-x-y}^{\infty}\int_{0}^{t}(1-\alpha)^{i-j}f_{X_1}(x)f_{X_2}(y)f_{X_3}(z)f(\tau)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau$$

(2) Opportunistic replacement is performed at $S_{A_n+m+1}+\tau$, after both the minor and severe defective stages are identified, the system is replaced at OM as shown in Figure 4 (b).

$$P(T_{or}=S_{A_n+m+1}+\tau)=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}p\left\{\begin{array}{l} S_{A_n+j}<X_1<S_{A_n+j+1} \\ i-j \quad \text{minor} \quad \text{inspections} \quad \text{didn't} \\ \text{identify} \quad \text{the} \quad \text{minor} \quad \text{defective} \quad \text{stage} \\ S_{A_n+m}-X_1<X_2<S_{A_n+m+1}-X_1 \\ X_3>S_{A_n+m+1}+\tau-X_1-X_2, \quad \tau\le t \end{array}\right\} \tag{10}$$

$$=\sum_{i=0}^{N_{n+1}-1}\sum_{j=0}^{i}\int_{S_{A_n+j}}^{S_{A_n+j+1}}\int_{S_{A_n+m}-x}^{S_{A_n+m+1}-x}\int_{S_{A_n+m+1}+\tau-x-y}^{\infty}\int_{0}^{t}\delta(i,j,N_{n+1})f_{X_1}(x)f_{X_2}(y)f_{X_3}(z)f(\tau)\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\tau$$

3.4. The Long Run Expected Cost per Unit Time

The probabilities of the system being replaced under different scenarios have been formulated. Then, the expected replacement cycle cost and the expected replacement cycle length of the system need to be expressed to obtain the long run expected cost per unit time.

In order to obtain the expected replacement cycle cost, the expected replacement cycle costs under different replacement scenarios are given first.

(1) When the maintenance action is corrective replacement, the expected replacement cycle cost $E(C_{cr})$ can be expressed as

$$E(C_{cr})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}-1}\left[(A_n+i-n)c_{mi}+nc_{ma}+c_{cr}\right]\times P(S_{A_n+i}<T_{cr}<S_{A_n+i+1})+\\ \sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}-1}\left[(A_n+i+1-h(i+1,A_n))c_{mi}+h(i+1,A_n)c_{ma}+c_{cr}\right]\times P(S_{A_n+i+1}<T_{cr}<S_{A_n+i+1}+\tau)+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[(A_n+m-h(m,A_n))c_{mi}+h(m,A_n)c_{ma}+c_{cr}\right]\times P(S_{A_n+m}<T_{cr}<S_{A_n+m+1})+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[(A_n+m+1-h(m+1,A_n))c_{mi}+h(m+1,A_n)c_{ma}+c_{cr}\right]\times P(S_{A_n+m+1}<T_{cr}<S_{A_n+m+1}+\tau)\end{array}\right\} \tag{11}$$

Where $h(x,A_n)$ is defined as

$$h(x,A_n)=\begin{cases}n, & A_n+x<A_{n+1}\\ n+1, & A_n+x=A_{n+1}\\ h(x,A_{n+1}), & A_n+x>A_{n+1}\end{cases} \tag{12}$$

Where $A_n=\sum_{k=0}^{n}N_n$ and $N_0=0$. $N_n(n=1,2,\cdots)$ is derived from $N=\{N_1,N_2,\cdots\}$. N is a set of sequential numbers, which mean how many minor inspections are implemented between two major inspections.

(2) When inspection replacement happens, the expected replacement cycle cost $E(C_{ir})$ is given by

$$E(C_{ir})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}-1}\left[(A_n+i+1-h(i+1,A_n))c_{mi}+h(i+1,A_n)c_{ma}+c_{ir}\right]\times P(T_{ir}=S_{A_n+i+1})+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[(A_n+m+1-h(m+1,A_n))c_{mi}+h(m+1,A_n)c_{ma}+c_{ir}\right]\times P(T_{ir}=S_{A_n+m+1})\end{array}\right\} \tag{13}$$

(3) If opportunistic replacement is implemented, the expected replacement cycle cost $E(C_{or})$ is

$$E(C_{or})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}-1}\left[(A_n+i+1-h(i+1,A_n))c_{mi}+h(i+1,A_n)c_{ma}+c_{or}\right]\times P(T_{or}=S_{A_n+i+1}+\tau)+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[(A_n+m+1-h(m+1,A_n))c_{mi}+h(m+1,A_n)c_{ma}+c_{or}\right]\times P(T_{or}=S_{A_n+m+1}+\tau)\end{array}\right\} \tag{14}$$

Then, the expected replacement cycle lengths of the system under different replacement scenarios are also formulated.

(1) When the maintenance action is corrective replacement, the expected replacement cycle length $E(L_{cr})$ can be expressed as

$$E(L_{cr})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}\text{-}1}\int_0^{T_{A_n+i+1}}\left[S_{A_n+i}+z\right]\times P(S_{A_n+i}+z<T_{cr}<S_{A_n+i}+z+\mathrm{dz})\mathrm{dz}+\\ \sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}\text{-}1}\int_0^{\tau}\left[S_{A_n+i+1}+z\right]\times P(S_{A_n+i+1}+z<T_{cr}<S_{A_n+i+1}+z+\mathrm{dz})\mathrm{dz}+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\int_0^{T_{A_n+m+1}}\left[S_{A_n+m}+z\right]\times P(S_{A_n+m}+z<T_{cr}<S_{A_n+m}+z+\mathrm{dz})\mathrm{dz}+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\int_0^{\tau}\left[S_{A_n+m+1}+z\right]\times P(S_{A_n+m+1}+z<T_{cr}<S_{A_n+m+1}+\mathrm{z}+\mathrm{dz})\mathrm{dz}\end{array}\right\} \tag{15}$$

(2) When inspection replacement happens, the expected replacement cycle length $E(L_{ir})$ is given by

$$E(L_{ir})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}\text{-}1}\left[S_{A_n+i+1}\right]P(T_{ir}=S_{A_n+i+1})+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[S_{A_n+m+1}\right]P(T_{ir}=S_{A_n+m+1})\end{array}\right\} \tag{16}$$

(3) If opportunistic replacement is implemented, the expected replacement cycle length $E(L_{or})$ is

$$E(L_{or})=\left\{\begin{array}{l}\sum_{n=0}^{\infty}\sum_{i=0}^{N_{n+1}\text{-}1}\left[S_{A_n+i+1}+\tau\right]P(T_{or}=S_{A_n+i+1}+\tau)+\\ \sum_{n=0}^{\infty}\sum_{m=i+1}^{\infty}\left[S_{A_n+m+1}+\tau\right]P(T_{or}=S_{A_n+m+1}+\tau)\end{array}\right\} \tag{17}$$

Based on the known expected replacement cycle cost and expected replacement cycle length of the system, the long run expected cost per unit time can be obtained.

$$c(\mathbf{T},\mathbf{N},t)=\frac{E(C_{cr})+E(C_{ir})+E(C_{or})}{E(L_{cr})+E(L_{ir})+E(L_{or})} \tag{18}$$

Where $T=\{T_1,\mathrm{T}_2,\cdots\}$ and $N=\{N_1,N_2,\cdots\}$. If the optimal T, N, and t can be solved, the minimum long run expected cost per unit time will be obtained. The method to search for optimal decision variables will be studied in future works.

4. Some Special Cases of this Model

For comparison, some simpler PM models are given. These simpler models are special cases of the proposed model, which can be presented by adopting specific parameters or decision variables. Furthermore, the generality of the proposed model is proven by discussing these special cases.

4.1. Periodic Inspection Model (*Model 1*)

First, the model proposed in literature [11] (Model 1) is introduced. The system is also subject to minor and major inspections. Both minor and major inspections are periodic inspections. The minor inspection interval is T before the defective stage is identified, i.e. $T_k=T,\ S_k=kT,\ (k=1,2,\cdots,i)$. Once the system is found in the minor defective stage at $iT(iT=S_i)$, the next minor inspection interval is cut to $T/2$, i.e. $T_k=T/2,\ (k>i)$. The major inspection interval is NT or $N(T/2)$, which is N times the minor inspection interval, i.e. $N_k\equiv N,\ (k=1,2,\cdots)$. The other assumptions are the same as the proposed model. T, N, and t are taken as decision variables for Model 1. In literature [11], the optimal solutions of Model 1 are solved: $T^*=4$, $N^*=3$, and $t^*=2$.

4.2. Sequential Inspection Model without Imperfect Inspection (*Model 2*)

Due to its low cost, imperfect inspection is adopted in this paper. The minor inspection is imperfect inspection, which can identify the minor defective stage with a certain probability but can detect both the normal and severe defective stages perfectly. Substituting perfect inspection for imperfect inspection, a sequential inspection model without imperfect inspection (Model 2) is obtained. In Model 2, inspections can always identify the state of the system perfectly and there is no difference in inspection cost, i.e. $\alpha = 1$ and $c_{mi} = c_{ma}$. The decision variables of Model 2 are T and t.

4.3. Sequential Inspection Model Without Postponed Maintenance and OM (*Model 3*)

When the threshold $t = 0$, postponed maintenance and OM will never happen, and a sequential inspection model without postponed maintenance and OM (Model 3) is obtained. For Model 3, only inspection replacement or corrective replacement is implemented. Once the system is found in the severe defective stage, it will be replaced immediately. Formulas of Model 3 can be derived by simplifying the equations established in Section 3. The decision variables of Model 3 are T and N.

5. Numerical Example

In this section, numerical examples are provided to explain the utility of this model. First, distributions of each stage of the system and parameters are given. Then, a comparison with other models proposed in Section 4 is made to demonstrate the economy of this model. Last, a simple parameters analysis is given to discuss the influence of parameters on the long run expected cost per unit time.

To make comparison easier, most of the pdfs and parameters are quoted from literature [11] in this numerical example. Accordingly, it is assumed that the three stages in the gradual degradation process follow three Weibull distributions, and their pdfs are

$$f_{x_n}(x;\lambda_n,k_n) = \frac{k_n}{\lambda_n}(\frac{x}{\lambda_n})e^{-(x/\lambda_n)^{k_n}} \tag{19}$$

Where λ_n are scale parameters and k_n are shape parameters $(n = 1,2,3)$. Parameters used in this numerical example are listed in Table 2.

Table 2. Parameters used in numerical example

λ_1	15.83	k_1	1.47
λ_2	9.61	k_2	1.95
λ_3	6.02	k_3	2.81
α	0.4	c_{mi}	8
c_{ma}	20	c_{cr}	3000
c_{ir}	400	c_{or}	200
T_{om}	200		

A set of specific decision variables are given to demonstrate the economy of this model. It is noted that these specific decision variables are not the optimal solutions. However, if it is proven that the proposed model is more cost-effective than other models even though the solutions are not the optimal ones, the economic advantage will be more obvious when optimal solutions are solved. Solving optimal solutions of the proposed model is a difficult problem that will be studied in future works.

It is supposed that T and N are two specific sequences, which satisfy the following conditions:

$T = \{T_1, T_2, \cdots, T_k, T_{k+1}, \cdots\}$, $(k = 1,2,\cdots)$:

$$T_{k+1} = \begin{cases} T_k \cdot T_q, & T_k \cdot T_q > T^0 \\ T^0, & T_k \cdot T_q \le T^0 \end{cases} \tag{20}$$

Where T_q and T^0 are predetermined constants. T_q can be considered the common ratio, and T^0 is the lower bound of this sequence. Once T_1, T_q, and T^0 are known, the whole sequence T is defined.

Similarly, $N = \{N_1, N_2, \cdots, N_k, N_{k+1}, \cdots\}$, $(k = 1, 2, \cdots)$:

$$N_{k+1} = \begin{cases} N_k - N_d, & N_k - N_d > N^0 \\ N^0, & N_k - N_d \le N^0 \end{cases} \tag{21}$$

Where N_d and N^0 are also predetermined constants. N_d can be considered the common difference, and N^0 is the lower bound of N. Once N_1, N_d, and N^0 are given, the whole sequence N is defined as well.

Based on adopting the above specific sequences, a comparison between the proposed model and others model is presented. Decision variables adopted in this example are given in Table 3. Model 1 takes the optimal solution that is solved in paper [11]. Aside from this, the decision variables of other models remain consistent with those of the proposed model. A simulation algorithm is designed to code the whole PM model, and the results under various minor inspection intervals are obtained by many enumerations.

Table 3. Decision variables adopted in example

	t	$N_1(N)$	N_d	T_q	N^0	T^0
Proposed model	2	10	1	0.8	3	2
Model1	2	3	--	--	--	--
Model2	2	--	--	0.8	--	2
Model3	0	10	1	0.8	3	2

As illustrated in Figure 5, the abscissa stands for the first item (T_1) of the minor inspection intervals sequence T (for the proposed model and Model 2) or the inspection interval T of Model 1. The ordinate is the long run expected cost per unit time. The result reveals that the proposed model is more cost-effective than others, even though T and N are not the optimal solutions. This economic benefit may result from the following facts: 1) In the early operation period, the system in the normal stage is a high probability event, and longer inspection interval can save costs. After a period of operation, shortening the inspection interval along with increasing the runtime could prevent unexpected failure. 2) The benefit of taking low-cost imperfect inspection is believed to outweigh the risk that the minor defective stage cannot be identified perfectly by imperfect inspection. The comparison between Model 3 and the proposed model is discussed in the next simulation, where the effect of T_{OM} and t on the long run expected cost per unit time is also presented.

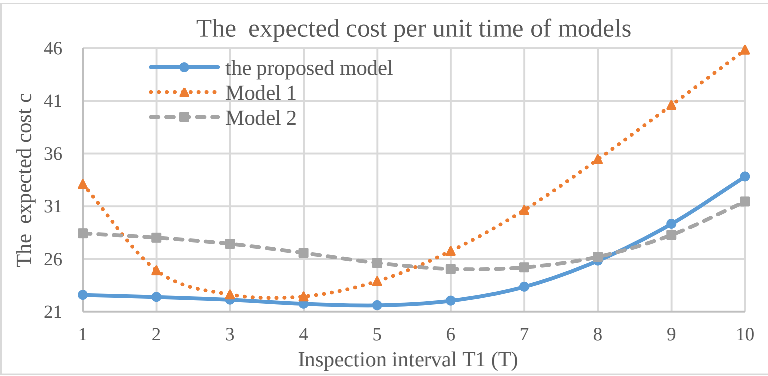


Figure 5. Comparison between the proposed model and other models

In Table 4, c^* is the minimum long run expected cost per unit time when T and N are the specific sequences. T_1^* is the first item of the specific minor inspection intervals sequence that minimizes the long run expected cost per unit time. When the threshold $t = 0$, the special case Model 3 is obtained. Comparing with the proposed model discussed in the previous example $(T_{OM}$ =200, $t = 2)$, it appears that introducing postponed maintenance and OM can generate a very limited economic benefit. However, the cycle length of OM is usually shorter than the expected life cycle length of the system in the practical industry. For example, the refueling cycle length of the reactor is about a year, which would lead to an OM for other equipment in the nuclear power plant. Generally speaking, the expected life cycle length of the equipment is usually longer than one year in nuclear power plants. $T_{OM}=200$ means the OM interval is much longer than the expected life cycle length of the system in this simulation. Therefore, a discussion is made to present the effect of T_{OM} on the long run expected cost per unit time. As Table 4 shows, the cost is reduced gradually as the OM interval decreases. It is rational to consider that introducing postponed maintenance and OM could be an effective method to improve the economic performance.

Table 4. The effect of T_{OM} and t on the expected cost

T_{OM}	t	T_1^*	c^*	T_{OM}	t	T_1^*	c^*
	0		21.6327	150		5	21.5905
	1		21.5926	100		5	21.5744
	2		21.5767	50		5	21.5149
200	3	5	21.6317	40	2	5	21.4559
	4		21.8111	30		5	21.4215
	5		22.0624	20		5	21.3141
	10		24.3215	10		4	20.8560

6. Conclusions

A PM model is proposed for a system subject to a three-stage failure process. In the proposed model, two-level sequential inspections, postponed maintenance and OM, are considered. It is assumed that the failure process of system can be divided into three independent stages: normal, minor defective, and severe defective stages. Considering that the inspection interval changing with the running time of the system would be more cost-effective, two-level sequential inspections are introduced. The minor inspection is a low-cost imperfect inspection that can identify the minor defective stage with a certain probability but can identify the other two stages completely. The major inspection is a perfect inspection that can distinguish the states of the system perfectly. When the system is found in the severe defective stage, it should be replaced immediately if the time to the next OM is greater than a threshold level; otherwise, the replacement is postponed to the next OM. Different replacement situations are studied, and the probabilities of them are derived. Based on this, the expected replacement cycle cost and expected replacement cycle length of the system are obtained, and thus the long run expected cost per unit time is known. Then, some other models are discussed, and these models are special cases of the proposed model and can be obtained by adopting specific parameters or decision variables. In the end, comparison with other models and simple parameters analysis are given to demonstrate the utility of this model in numerical examples. The simulation results demonstrate that there is at least a set of solutions that make the proposed model more economical than other simpler models, even though the solutions are not the optimal ones.

There are several further research topics left for future works based on this paper. First, the optimal solution can be searched to acquire more economic benefits. Second, if the system is found in the minor defective stage by either inspection, more actions can be taken to improve the economic performance, such as preparing maintenance in advance to reduce maintenance cost. Third, the long run expected cost per unit time is selected as a measure to optimize decision variables in this paper. However, availability is also a remarkable measure besides cost in practice. Therefore, the proposed model could be translated to a new model that can be measured in terms of availability.

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