

# Dynamic Time Series Reliability Analysis for Long-Life Mechanic Parts with Stress-Strength Correlated Interference Model

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## Abstract

Based on data of the equivalent stress from the ANSYS for a loaded hollow shaft, the correlation between a mechanical part's elastic modulus and the corresponding Von Mises stress is statistically verified in this paper. Using the Copula correlation theory, a static reliability model involving stress-strength interference is built. According to the performance degradation data of mechanical parts with long-life and high-reliability, deterministic time series models are used to extract the characteristic information of the distribution of degradation variables, and then a method is proposed for estimating the characteristic parameters of degradation strength and integrated stress. Two-stage maximum likelihood estimation is applied to determine the scalar degree of correlation between both, and then a reliability assessment of long-life mechanical parts is completed.

**Keywords:** stress-strength interference model; time series; correlation; copula; reliability; statistical analysis

(Submitted on October 5, 2018; Revised on November 21, 2018; Accepted on December 16, 2018)

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## 1. Introduction

In 1947, Freudenthal [1] proposed the famous stress-strength interference model, which formed the basis of quantitative analysis design of mechanical component reliability. In the stochastic probabilistic reliability analysis method, the stress and strength are usually assumed as static random variables that are independent of each other [2-7]. In practice, due to the long-term effects of mechanical parts under dynamic load, fatigue, wear, corrosion, aging, and other factors cause component strength monotone attenuation. On the other hand, due to crack initiation, deepening of corrosion degree, and changes of dimension gap, the stress value of the force point changes with time. In addition, it is possible to be subjected to random impact stress, which results in a random process rather than constant [8-11]. Therefore, the dynamic analysis of stress-strength double random process interference is more practical for the reliability design of mechanical parts.

The allowable load (comprehensive strength) for mechanical parts usually increases with the increase in size parameters of the parts, material physical properties, and good surface quality factor. Meanwhile, parts of the stress of the focus of variable (consolidation stress) change because of the same kinds of factors. Thus, the stress-strength of the parts is correlated. The reliability calculation of independent interference is only a simplified approximation of such characteristics, and the correlation failure is the universal characteristic of mechanical reliability engineering. It is also a key issue to be solved urgently [12-16]. The stress-strength correlation interference model considers the correlation and is more widely applied.

For high reliability and long-life products, using traditional life tests and even accelerated life tests is often difficult to obtain the full life span of the data samples within the given time, which causes the classical failure statistical methods to lose efficacy. Fatigue, corrosion, aging, and other degradation characteristics are typically presented in the failure process of high reliability and long-life products. During the test, the degradation parameters of this kind of products are tested, and the degraded data is obtained while the statistical analysis is also completed, which is an effective and cost-saving reliability evaluation method [17-24].

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In this paper, the finite element analysis software ANSYS is used to calculate the equivalent stress of a loaded hollow shaft, and the node Von Mises stress value is extracted to verify the correlation between the machinery of the hollow shaft Von Mises stress and parts comprehensive strength component (modulus of elasticity). Then, based on the Copula function, the reliability calculation model of stress-strength correlation is built. Lastly, with the performance degradation data of high long-life mechanical products, a deterministic trend combination time sequence model is adopted to give the reliability prediction method for the product reliability under the dynamic interference of stress-intensity correlation.

## 2. Model Assumptions and Symbols

For high reliability and long-life mechanical parts, the test results are obtained by testing the parameters (such as crack length, corrosion depth, abrasion degree, etc.) of the machine parts. Assuming that:

(1)  $n$  samples were tested and the interval time was the same. The test time is denoted as  $t_0, t_1, \dots, t_m$ .

(2) The comprehensive strength of the part under load time  $t$  is  $\{\delta(t), t \in T\}$ . The density function of  $\delta(t)$  is  $f_{\delta}(\delta, \beta(t))$ , and the corresponding distribution function is  $F_{\delta}(\delta, \beta(t))$ .  $\beta(t)$  is the distributed characteristic parameter, which in normal distribution is  $\beta(t) = (\mu_{\delta}(t), \sigma_{\delta}(t))$ .

(3) The combined stress of the parts is  $\{S(t), t \in T\}$ , in which the density function of  $S(t)$  is denoted as  $g_s(s, \alpha(t))$ , and the distribution function as  $G_s(s, \alpha(t))$ .  $\alpha(t)$  is the distributed characteristic parameter.

(4) The related structure of comprehensive strength  $\delta$  and comprehensive stress  $S$  is expressed by Copula  $C_{\theta}(u, v)$ , in which  $\theta$  is the relative degree parameters of the two. The Gumbel Copula and the Clayton Copula are common positive/negative Copula families in engineering.

$$C^{Gum}(u, v; \theta) = \exp(-[(-\ln u)^{\frac{1}{\theta}} + (-\ln v)^{\frac{1}{\theta}}]^{\theta}), \theta \in (0, 1] \quad (1)$$

$$C^{Cla}(u, v; \theta) = \max\{(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, 0\}, \theta \in [-1, 0) \quad (2)$$

It can be proved by the limit theory and by Robida's law that when  $\theta \rightarrow 1$ , the variables described in  $C^{Gum}$  tend to be independent of each other; when  $\theta \rightarrow 0$ , the variables described in  $C^{Gum}$  tend to be perfect positive correlation; when  $\theta \rightarrow -1$ ,  $C^{Cla}$  reflects a completely negative correlation; and when  $\theta \rightarrow 0$ ,  $C^{Cla}$  tends to be independent.

(5) The performance test data of the  $i^{\text{th}}$  test time  $t_j$  is  $x_i(t_j)$ , ( $i = 1, 2, \dots, n$ ;  $j = 1, 2, \dots, m$ ).

## 3. Statistical Analysis of Correlation Interference Between Stress-Strength Components

In this section, a large finite element analysis software ANSYS is applied for the stress calculation of mechanical parts. The model consists of a hollow shaft and a disk. The diameter of the disc is 200mm, the thickness is 25mm, the outside/inside diameter of the hollow shaft is 35mm/25mm, and the length is 150mm. The model adopts the SOLID45 unit, with a total of about 5040 units and 6720 nodes. The finite element model is shown in Figure 1. The hollow shaft and the disc are connected under an interference fit, and the interference amount is 0.04mm. The nonlinear analysis was calculated by using ANSYS, and the load was applied to the axial end with a weight of 120N. The materials used in the finite element calculation are CL60 steel, class B wheel steel, aluminum, and other 20 materials, as shown in Table 1.

The hollow shaft components of the disk are constructed by using the 20 materials in Table 1. The disc section was extracted by ANSYS. As shown in Figure 2, the results of Von Mises stress at node 7 are shown in Table 2.

SPSS is utilized to analyze the elastic modulus of the material and the calculated Von Mises stress value. Moreover, the distribution was fitted and the characteristic parameter values were obtained. The statistical results of the characteristic parameters are shown in Table 3.

Based on the comparison of normal distribution, uniform distribution, logarithmic normal, and exponential distribution, Table 4 shows the optimal approximate distribution of SPSS fitting.

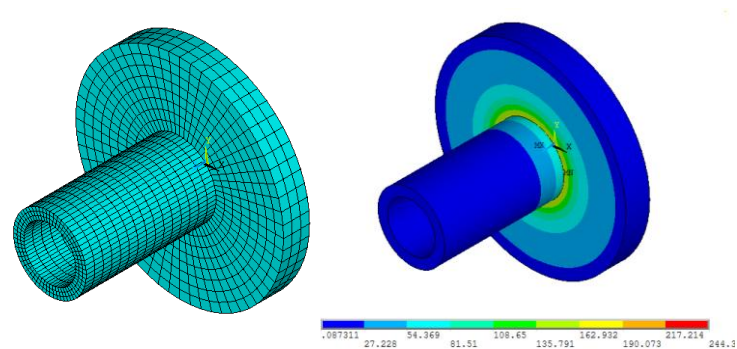


Figure 1. ANSYS computing model of hollow shaft and disk assembly

Table 1. The elastic modulus of 20 materials used in the parts

No.	Material	Modulus of elasticity/GPa
1	Cl60 steel	205
2	B-wheel steel	192
3	45# steel	206
4	Aluminum	71.7
5	Porcelain	55
6	Lead	17
7	Gray cast iron	130
8	Cast aluminum bronze	105
9	Cold-drawing brass	95
10	Rolling zinc	84
11	Cast iron	100
12	Stainless steel	190
13	Magnesium	44.8
14	Nickel	207
15	Glass	46.2
16	Graphite	36.5
17	Titanium	102.04
18	Tungsten	344.7
19	Wood	11
20	Rubber	0.00784

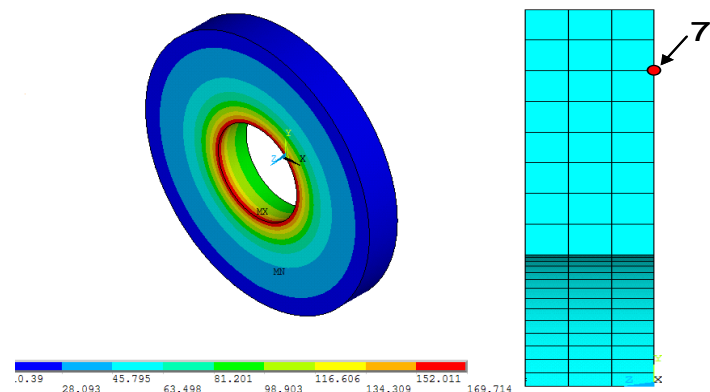


Figure 2. Finite element calculation of disk ANSYS

Table 2. Elastic modulus of 20 materials used in the parts

No.	Material	Von Mises/MPa
1	Cl60 steel	13.98746
2	B-wheel steel	12.9309
3	45# steel	13.83837
4	Aluminum	4.94067
5	Porcelain	3.64025
6	Lead	1.20912
7	Gray cast iron	8.84996
8	Cast aluminum bronze	7.0457
9	Cold-drawing brass	6.22824
10	Rolling zinc	5.66791
11	Cast iron	6.62864
12	Stainless steel	12.95731
13	Magnesium	3.09795
14	Nickel	14.06966
15	Glass	3.08982
16	Graphite	2.5808
17	Titanium	6.94558
18	Tungsten	23.39905
19	Wood	0.75896
20	Rubber	0.00284

Table 3. The elastic modulus of 20 materials and the corresponding Von Mises stress distribution parameters

	Modulus of elasticity	Von Mises stress
N	20	20
Mean	112.15	7.59346
Std. Deviation	87.665	5.94303
Minimum	0	0.00284
Maximum	345	23.39905

Table 4. The elastic modulus of 20 materials and SPSS of corresponding Von Mises stress were fitted

The fitting of exponential distribution		Modulus of elasticity	Von Mises stress
N		20	20
Exponential parameter	Mean	112.15	7.5935
Most extreme differences	Absolute	.129	.138
	Positive	.108	.107
	Negative	-.129	-.138
Kolmogorov-smirnov z		.578	.618
Asymp. Sig. (2-tailed)		.892	.840

With the data of statistical samples in Tables 1 and 2, set the parts material elastic modulus for  $X$  and the corresponding ANSYS to calculate the Von Mises stress for  $Y$  to validate the mechanical parts of comprehensive strength, the modulus of elasticity for the intensity component, and the correlation between the parts under stress. The joint scatter of  $X$  and  $Y$ , i.e.,  $(x_i, y_i), (i = 1, 2, \dots, 20)$ , is shown in Figure 3.

According to the correlation between material elastic modulus  $X$  described in Figure 3 and the corresponding Von Mises stress  $Y$ , and combined with the fitting density function,  $f_X(x), g_Y(y)$ , shown in Table 4, the Gumbel copulas is used to build its related structure. It estimates the related degree of maximum likelihood for the result where  $\hat{\theta} \approx 0.05$ . The random simulated scatter diagram of copula  $C^{Gum}(u, v; \hat{\theta} = 0.05)$  is shown in Figure 4.

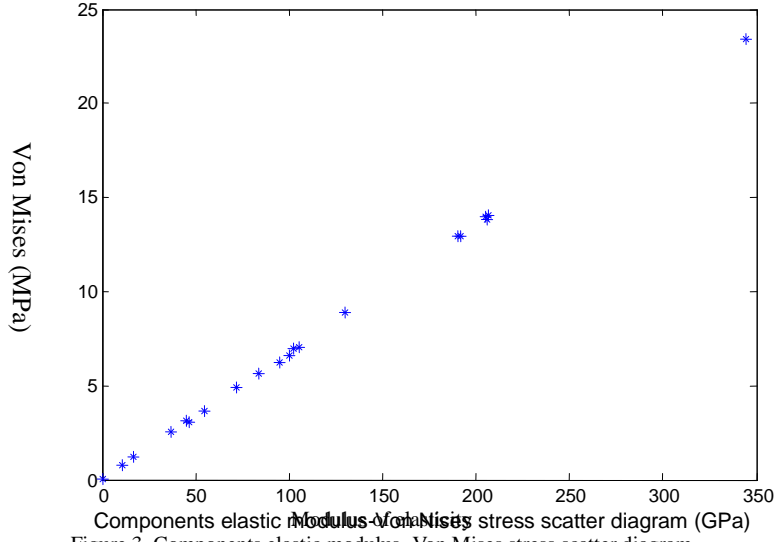
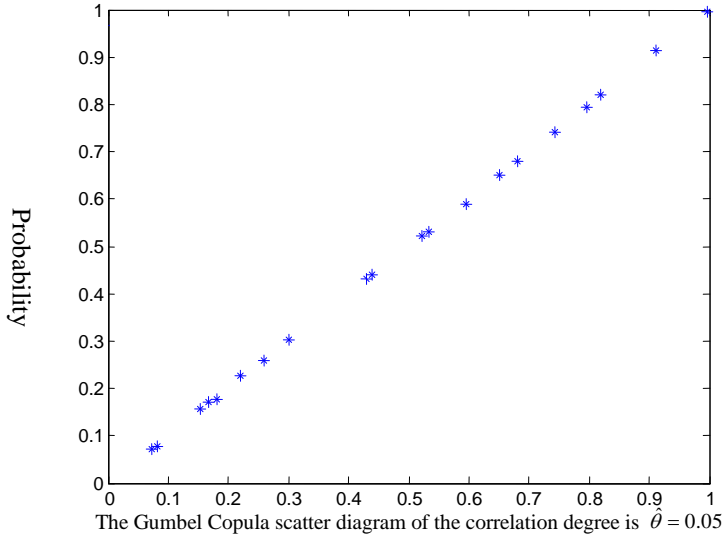


Figure 3. Components elastic modulus - Von Mises stress scatter diagram

Figure 4. The Gumbel Copula scatter diagram of the correlation degree is  $\hat{\theta} = 0.05$ 

Similarly, the yield strength, tensile strength, shear strength, and other strength components of the 20 materials were analyzed and fitted to determine the density function of the component comprehensive strength  $\delta(t)$  as  $f_{\delta_i}(\delta, \mathbf{\beta}(t))$ , and the comprehensive stress density function as  $g_{s_i}(s, \mathbf{\alpha}(t))$ . Also, the Copula selection and the correlation degree parameter estimation of  $\hat{\theta}$  were completed.

#### 4. The Component Reliability Calculation Model based on the Stress-Strength Correlation

In mechanical reliability design, the traditional stress-strength interference model assumes that the integrated stress  $S$  and the comprehensive strength  $\delta$  are independent of each other. The classical parts reliability prediction model is

$$R = P(\delta > S) = \int_{-\infty}^{+\infty} g(s) \cdot \int_s^{+\infty} f(\delta) d\delta ds \quad (3)$$

The correlation between  $\delta$  and  $S$  is presented that for an arbitrary  $(x, y) \in R^2$ ,

$$P(\delta > x, S \leq y) \geq P(\delta > x)P(S \leq y) \quad (4)$$

The corresponding structure is characterized by Copula

$$C_{\theta}(F_{\delta}(x), G_s(y)) \leq F_{\delta}(x) \cdot G_s(y) \quad (5)$$

The Sklar theorem [25] guarantees the existence and uniqueness of the Copula  $C_{\theta}$  in any two-dimensional random variable  $(X, Y)$ .

**Theorem 1** [25] Let the marginal distribution function of the two-dimensional random variable be  $F(\cdot)$ ,  $G(\cdot)$ . Then, there exists only the two-dimensional Copula  $C_{\theta}(u, v)$ , which makes it arbitrary  $x, y \in \bar{R}$ ,

$$P(X \leq x, Y \leq y) = C_{\theta}(F(x), G(y)) \quad (6)$$

If the correlation between  $X$  and  $Y$  is not considered, this means they are independent of each other. Then,

$$C_{\theta}(u, v) = uv \quad (7)$$

In this situation, the reliability of the parts under stress  $S$  and strength  $\delta$  is

$$\begin{aligned} R &= P(\delta > S) = \int_{-\infty}^{+\infty} g(t) \cdot P(\delta > t \mid S = t) dt \\ &= \int_{-\infty}^{+\infty} g(t) \cdot \left[ \int_t^{+\infty} f_{\delta|S=t}(\delta \mid s) d\delta \right] dt \\ &= \int_{-\infty}^{+\infty} g(s) \cdot \left[ \int_s^{+\infty} \frac{\partial^2 C_{\theta}(u, v)}{\partial u \partial v} \cdot f(\delta) \Big|_{\substack{u=F(\delta) \\ v=G(s)}} d\delta \right] ds \end{aligned} \quad (8)$$

In the situation that the integrated stress and comprehensive strength of the parts are independent,  $\frac{\partial^2 C_{\theta}(u, v)}{\partial u \partial v} = 1$ , and then

$$R = P(\delta > S) = \int_{-\infty}^{+\infty} g(s) \cdot \int_s^{+\infty} f(\delta) d\delta ds \quad (9)$$

The results are consistent with the classical independent interference reliability calculation model. In fact, an independent relationship is only a special case of correlation, and the independent interference model is only a simplified special case of the correlation interference model.

## 5. Dynamic Timing Reliability Statistical Analysis of Long-Life Mechanical Parts

### 5.1. Dynamic Timing Estimation of Characteristic Parameters of Degradation Strength Distribution

According to the performance degradation data  $x_i(t_j)$  samples of  $t_j$  at the time of detection, the dynamic strength samples,  $\delta_i(t_0), \delta_i(t_1), \dots, \delta_i(t_m)$ , of the parts are transformed based on the method of strength determination [26-27]. The distribution type  $f_{\delta_i}(\delta)$  is determined by non-parametric distribution fitting. In most working conditions, the normal distribution is the most common situation for the random diversity of the interfering factors, so the normal distribution is chosen to be related.

The estimation of distribution characteristic parameters  $\beta(t)$  is realized with maximum likelihood estimation:

$$\begin{aligned} L(\beta(t_j)) &= L(\mu_{\delta}(t_j), \sigma_{\delta}(t_j)) \\ &= \prod_{i=1}^n f_{\delta_i}(\delta_i(t_j), \mu_{\delta_i}(t_j), \sigma_{\delta_i}(t_j)) \end{aligned} \quad (10)$$

By solving the likelihood equations, we get the parameter vector estimation sequence of  $(t = t_0, t_1, \dots, t_m)$  and the moment component strength  $\hat{\beta}(t) = (\hat{\mu}_\delta(t), \hat{\sigma}_\delta(t))$ , in which

$$\{\hat{\mu}_\delta(t)\} = \{\hat{\mu}_\delta(t_0), \hat{\mu}_\delta(t_1), \dots, \hat{\mu}_\delta(t_m)\}, \{\hat{\sigma}_\delta(t)\} = \{\hat{\sigma}_\delta(t_0), \hat{\sigma}_\delta(t_1), \dots, \hat{\sigma}_\delta(t_m)\}$$

Due to the long-term dynamic load of mechanical parts, its strength  $\{\delta(t)\}$  is attenuated randomly. The dynamic components  $\{\hat{\mu}_\delta(t)\}$  and  $\{\hat{\sigma}_\delta(t)\}$  of the characteristic parameter vector  $\hat{\beta}(t)$  show the deterministic trend sequence. For this purpose, the dynamic effect of the fitting intensity parameters of the quadratic mean trend model is established:

$$\begin{cases} \hat{\mu}_\delta(t) = c_0 + c_1 t + c_2 t^2 + \mu(t) \\ \hat{\sigma}_\delta(t) = d_0 + d_1 t + d_2 t^2 + \sigma(t) \end{cases} \quad (11)$$

Where  $c_i, d_i, (i=0,1,2)$  is the parameter of undetermined trend, which can be processed by standard regression analysis. First, the concrete form is fitted, namely, based on the least squares estimation method. With the empirical samples  $\{\hat{\mu}_\delta(t_0), \hat{\mu}_\delta(t_1), \dots, \hat{\mu}_\delta(t_m)\}$  and  $\{\hat{\sigma}_\delta(t_0), \hat{\sigma}_\delta(t_1), \dots, \hat{\sigma}_\delta(t_m)\}$ , the determination of the estimated value  $\hat{c}_i, \hat{d}_i$  is completed.  $\mu(t)$  and  $\sigma(t)$  are the wide stationary process of the zero mean, and then the residual  $\{\hat{\mu}_\delta(t) - \mu(t)\}$  and  $\{\hat{\sigma}_\delta(t) - \sigma(t)\}$  are analyzed and modeled in the stationary process. Through the model (12), the dynamic prediction of the characteristic parameters  $\hat{\mu}_\delta(t_d)$  and  $\hat{\sigma}_\delta(t_d)$  of the degradation intensity  $\{\delta(t_d)\}$  of the high long-life machine parts in the future  $t_d$  is completed. Thus, the distribution density function  $f_{\delta_{t_d}}(\delta, \hat{\beta}(t_d))$  of the component strength in  $t_d$  time is determined.

For the dynamic distribution density function  $g_{s_j}(s, \mathbf{a}(t))$  of the time stress course of the parts under the load, the same can be obtained according to the above time  $\{S(t)\}$  sequence statistical methods, which will not be repeated here.

## 5.2. Calculation of Dynamic Coherence Reliability of Stress-Strength and Estimation of Related Degree Parameters

Combined with stress-strength interference correlation of the static reliability calculation model (9) and the above section of part stress, a statistical method to determine the dynamic distribution of intensity and then the high long-life part time dynamic reliability model is expected in the future time  $t_d$ .

$$\begin{aligned} R(t_d) &= P(\hat{\delta}(t_d) > \hat{S}(t_d)) \\ &= \int_{-\infty}^{+\infty} g_{s_{t_d}}(s, \hat{\mathbf{a}}(t_d)) \cdot \left[ \int_s^{+\infty} \frac{\partial^2 C_{\theta_{t_d}}(u, v)}{\partial u \partial v} \cdot f_{\delta_{t_d}}(\delta, \hat{\beta}(t_d)) \Big|_{\substack{u=F_{\delta_{t_d}}(\delta, \hat{\beta}(t_d)) \\ v=G_{s_{t_d}}(s, \hat{\mathbf{a}}(t_d))}} d\delta \right] ds \end{aligned} \quad (12)$$

For the determination of the correlation degree  $\theta_{t_d}$  of the components' comprehensive stress process  $\{S(t_d)\}$  and the degradation intensity  $\{\delta(t_d)\}$  in the correlation interference model, it can be based on the empirical sample  $\delta_i(t_j), s_i(t_j)$ ,  $(i=1, 2, \dots, n; j=1, 2, \dots, m)$ , and the second phase maximum likelihood estimation obtains  $\hat{\theta}_{t_j}$ .

$$L(\theta_{t_j}) = \prod_{i=1}^n [c_{\theta_{t_j}}(u_1, u_2, \dots, u_p) \cdot \prod_{k=1}^p f_{X_k}(x_{ik}(t_j), \hat{\mu}_{X_k}(t_j), \hat{\sigma}_{X_k}(t_j))] \quad (13)$$

Where  $c_{\theta_{t_j}}$  and  $C_{\theta_{t_j}}$  are the density function, i.e.,  $c_{\theta_{t_j}} = \frac{\partial C_{\theta_{t_j}}(u_1, u_2, \dots, u_p)}{\partial u_1 \partial u_2 \dots \partial u_n} \Big|_{u_k=F_{X_k}(x_{ik}(t_j), \hat{\mu}_{X_k}(t_j), \hat{\sigma}_{X_k}(t_j))}$ .

In most conditions, due to the size of the parts abrasion, poor surface quality, aging, corrosion, fatigue, and other reasons, the stress-strength interference of A related degree showed  $\theta_i$  trend of average unabated by considering the effect of dynamic load. The exponential smoothing method is used to predict the timing sequence  $\{\hat{\theta}_{t_j}\}$ , and  $\hat{\theta}_{t_d}$  is obtained.

$$\hat{\theta}_{t_j} = a\hat{\theta}_{t_j} + (1-a)\hat{\theta}_{t_{j-1}} \quad (14)$$

Where  $0 < a < 1$  is the smoothing coefficient, whose value is determined by the least square regression of the sample sequence and in most cases satisfies  $0.05 < a < 0.3$ .

## 6. Examples

There is a linkage mechanism in a certain machine, which is under tension when working. The tension follows the normal distribution, where the mean value and standard deviation are  $\bar{F}=35\text{kN}$  and  $\sigma_F=3.66\text{kN}$  respectively. The material of the linkage mechanism is A5 steel, and its tensile strength follows the normal distribution, where the mean value and standard deviation are 50MPa and 4MPa respectively. The cross-section area of the linkage mechanism is  $896\text{mm}^2$ . At this situation, we need to analyze the reliability of the linkage mechanism.

The mean and standard deviation of tensile stress of the mechanical linkage are:

$$\bar{S} = \frac{\bar{F}}{A} = \frac{35 \times 10^3 \text{ N}}{896 \times 10^{-6} \text{ m}^2} = 39.1 \text{ MPa}$$

$$\sigma_s = \frac{\sigma_F}{A} = 4.08 \text{ MPa}$$

The correlation structure between tensile stress  $S$  and tensile strength  $\delta$  conforms to the Clayton model  $C_\theta(u, v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$ . The probability density function and cumulative probability distribution of  $S$  are:

$$g(s) = \frac{1}{\sqrt{2\pi} \times 4.08} \exp\left(-\frac{(s-39.1)^2}{2 \times 4.08^2}\right) \quad (15)$$

$$u = G(s) = \int_0^s g(x) dx \quad (16)$$

The probability density function and cumulative probability distribution of tensile strength  $\delta$  are:

$$f(\delta) = \frac{1}{\sqrt{2\pi} \times 4} \exp\left(-\frac{(\delta-50)^2}{2 \times 4^2}\right) \quad (17)$$

$$v = F(\delta) = \int_0^\delta f(x) dx \quad (18)$$

Then, by substituting the above two formulas into the stress-strength interference reliability model, the initial reliability  $R$  of the mechanical linkage is obtained with the change of the correlation degree parameters  $\theta$ , as shown in Figure 5.

From Figure 5, it can be seen that the link reliability calculated by stress  $S$  and strength  $\delta$  correlation interference is between  $[0.9064, 0.9719]$ , and it is a continuous value changing with the correlation degree parameter  $\theta$ .

$R = 0.9064$  at the left end of the interval is the reliability value when the stress  $S$  and the strength  $\delta$  are completely inversely correlated linear functions ( $\theta = -1$ ). If the traditional interference model (10) is adopted according to the independence theory, the reliability is calculated as the right endpoint value of the interval,  $R = 0.9719$ .



Obviously, the traditional interference model supposes that stress and strength are independent, which tends to open up the calculation results and may lead to overestimation of product reliability.

If the Monte Carlo method is adopted to simulate 4000 times by computer, the simulated sample value is substituted into the maximum likelihood estimation Equation (13), and the correlation degree parameter is obtained as -0.3517. At this point, the correlation structure of stress  $S$  and strength  $\delta$  is shown in Figure 6, and the reliability of the mechanical linkage can be obtained from (8) of  $R = 0.9512$ .

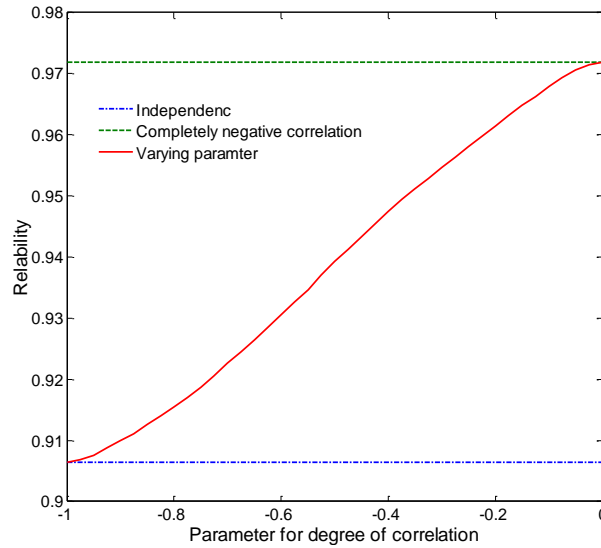


Figure 5. Time-varying reliability of the mechanical linkage

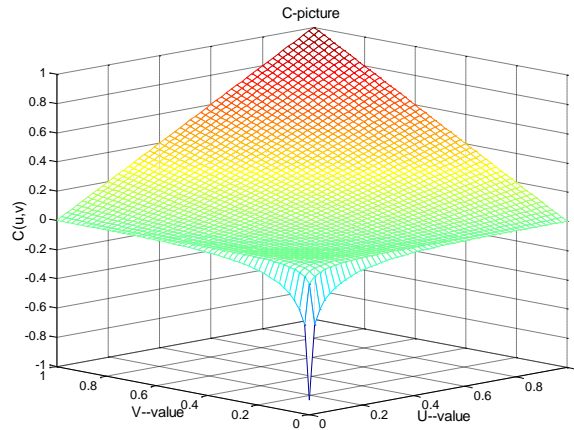


Figure 6. Stress-strength related structures Clayton copula model when  $\theta = -0.3514$

## 7. Conclusions

The analysis of correlation between stress and strength is the theoretical basis of reliability calculation of mechanical parts. Based on this, the reliability calculation model of stress-strength correlation interference is a generalization of the classical simplified model and becomes more suitable for practical applications.

The statistical theory of time series is an effective method for reliability analysis of long-life products. Based on the long-life and high mechanical parts performance degradation data using the deterministic trends combined time-series model, the comprehensive stress process of parts dynamic degradation characteristics parameter estimation method can be given. Based on two-stage maximum likelihood estimation, the comprehensive stress and comprehensive strength correlation degree are determined to complete the reliability evaluation of the stress-intensity correlation interference of long-lived parts. However, parameter estimation, correlation structure determination, and distribution prediction are all dependent on statistical analysis and information extraction of performance detection data. In general, the larger the sample size is, the more accurate the calculation results will be. Moreover, compared with the classical simplified model, this model is more computationally intensive.

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