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Reliability Assessment of Non-Repairable *k*-Out-Of-*n* System using Belief Universal Generating Function

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Abstract

Some research has been developed to handle aleatory and epistemic uncertainty in various engineering systems. But in this paper, we have established two methods, namely mass distribution and fuzzy reliability theory, to deal with these uncertainties in non-repairable *k*-out-of-*n*: G (F) systems, which has not been seen in the past. In the presented methodology, the failure rate (λ) is taken as a trapezoidal fuzzy number. Using the trapezoidal fuzzy number, expression for α-cut of fuzzy failure rate of every component and corresponding fuzzy reliability function has been calculated. Then, masses to the components are distributed with the help of these fuzzy reliability functions. By using these masses, reliability and MTTF of the considered systems have been computed. At last, a numerical example is taken to demonstrate the present approach.

Keywords: *k*-out-of-*n* system; fuzzy exponential distribution; belief universal generating function; system reliability; mean time to failure

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1. Introduction

The reliability analysis of a system, which is assembled to perform a certain function, plays an important role in day to day life as well as in designs of systems. System reliability is referred to in terms of a probability of its components. In order to ensure system reliability, involvement of engineers with reliability theory is necessary and is designated as reliability engineering. Reliability engineering is used to develop methods and tools to evaluate and demonstrate reliability, maintainability, availability and safety of components, equipment, and systems. Today consumers expect to have complex systems not only free from defects and systematic failures at time t=0, but also able to perform the required function failure-free for a stated time interval and have a fail-safe behavior in the case of critical or catastrophic failures. For these purposes, engineers used the concept of redundancy i.e., to improve the reliability of the system redundancy is applied. Due to the redundant nature of *k*-out-of-*n* configuration, it has gained popularity amongst reliability engineers.

k-out-of-*n* configuration may be classified into two main groups, namely *k*-out-of-*n*: F system and *k*-out-of-*n*: G system. If the failure of a system with n components is abandoned to the failure of at least k components ($k \leq n$), then the system is called *k*-out-of-*n*: F system. On the other hand, if the working of the system is abandoned to the working of at least k components ($k \leq n$), then the system is called *k*-out-of-*n*: G system. The *k*-out-of-*n* system structure is a commonly used redundancy technique and has a wide range of applications in aerospace, nuclear power, airborne weapon systems, computing and communication systems [23]. Considerable research efforts have been expanded in the reliability evaluation of binary *k*-out-of-*n* systems with identical/non-identical components [4,5,8,11,13,16]. There are several efficient algorithms available for computing the reliability of a non-repairable *k*-out-of-*n* system having different types of components. For details, one can refer [2,3,14,23,27]. Due to the variation among the importance of system's components, *k*-out-of-*n* configuration is further extended in terms of weight, and this extension is termed as weighted *k*-out-of-*n* system. This system is introduced by [12] and later expanded by [21,24,25].

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On the other hand, there are systems where different types of uncertainties about the state probabilities and performance rates of components [6] need to be modelled. There are different ways of classifying uncertainty, but one of the most widely used is to divide it into two types: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty comes from random processes. It is an irreducible and inherent uncertainty due to probabilistic variability. Epistemic uncertainty is reducible and is encountered due to lack of knowledge. Over the last few years, the reliability community has been increasingly aware that distinguishing between these types of uncertainty is important when evaluating the reliability of systems [18,19,22]. In order to remove uncertainties presented within the system, several methods have been proposed, some of which are interval approach, belief functions theory [5], possibility theory and fuzzy sets [17], fuzzy exponential distribution [7], fuzzy Rayleigh distribution [14], etc.

The universal generating function (UGF) technique first introduced by [10] and greatly extended by [1,9,21] is an efficient tool to evaluate the availability of different types of system. The UGF extends the moment-generating function and reduces the computational complexity of the MSS reliability assessment. Thanks to its efficiency, the UGF technique is also suitable for solving different system reliability optimization problems, as it can quickly evaluate system reliability. But in the case of uncertainty, classical UGF is not efficient enough to access the reliability of the system. To remove these uncertainties, it collaborated with different approaches, such as fuzzy set theory and belief theory, and reformulated as fuzzy UGF [26] and belief UGF [20] respectively. To get rid of this situation, Meeenakshi and Singh [15] also introduced probability intervals with hybrid UGF.

On the face of above studies, it is clear that in the past, reliability characteristics such as reliability and mean time to failure of non-repairable *k*-out-of-*n* systems are never obtained. Hence, in this study, belief UGF and fuzzy exponential distribution approaches are jointly used to evaluate the reliability of the proposed system. As a result, the proposed method is different from earlier discussed methods. So, the main focus of this paper is to evaluate the reliability characteristics of target systems with the help of belief universal generating function and fuzzy exponential distribution.

2. Assumptions

In the present model, the following assumptions are taken into account.

i) Initially, the system is in good condition.
ii) At t=0, all the components are perfectly well and at $t > 0$, they start operating.
iii) All the components are either working or failed.
iv) Failure rates of different components are supposed to be different.

3. **Notations**

BUGF	:	Belief Universal Generating function
MTTF	:	Mean Time to failure
G_j	:	Component j
g_j	:	State of the component j
$\tilde{\lambda}$	:	Fuzzy failure rate
$\tilde{p}_j$	:	Fuzzy probability of component j
G	:	Power set
λ_{jL}	:	Lower bound of failure rate of component j
λ_{jU}	:	Upper bound of failure rate of component j
p_{jL}	:	Lower bound of fuzzy probability component j
p_{jU}	:	Upper bound of fuzzy probability of component j
m_{ji_j}	:	Mass over the power set of G
X	:	Number of failed components
Y	:	Number of working components
$U_j(z)$	:	Belief u-function of component j
MTTF	:	mean time to failure

4. Preliminaries

4.1. Fuzzy exponential distribution

If the lifetime of a component (X) is modelled by an exponential distribution then

$$f(x;\lambda) = \lambda e^{-\lambda x}$$

where x is any variable and λ (the failure rate of the component) is crisp. Now, in order to have fuzzy exponential distribution, consider fuzzy number $\tilde{\lambda}$ in place of λ in the exponential distribution. In this case, the fuzzy probability of obtaining a value in the interval $[c, d],\ c \geq 0$ is $\tilde{p}(c \leq x \leq d)$ and its α-cut is as follows:

$$\tilde{p}(c \leq x \leq d)[\alpha] = \{\int_c^d \lambda e^{-\lambda x} dx \mid \lambda \in \tilde{\lambda}[\alpha]\} = [p^L[\alpha], p^U[\alpha]] \quad \forall \quad \alpha$$

where

$$p^L[\alpha] = \min\{\int_c^d \lambda e^{-\lambda x} dx \mid \lambda \in \tilde{\lambda}[\alpha]\} \text{ and } p^U[\alpha] = \max\{\int_c^d \lambda e^{-\lambda x} dx \mid \lambda \in \tilde{\lambda}[\alpha]\}$$

4.2. Fuzzy Reliability functions

Reliability or survival function $S(t)$ is the probability a unit survives beyond time t. Let the random variable X denote the lifetime of components, which has density function $f(x,\theta)$ (lifetime density function) and cumulative distribution function $F_X(t) = P(x \leq t)$. Then, the reliability at time t is defined as:

$$S(t) = P(x > t) = 1 - F_X(t), t > 0$$

If the lifetime is supposed to have fuzzy exponential distribution, then we replace λ with fuzzy number $\tilde{\lambda}$. If $\tilde{\lambda}$ is a trapezoidal fuzzy number, say (a_1, a_2, a_3, a_4), then its membership $\mu_{\tilde{\lambda}}(x)$ is given by

$$\mu_{\tilde{\lambda}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1} & , a_1 \leq x < a_2 \\ 1 & , a_2 \leq x < a_3 \\ \dfrac{x - a_3}{a_4 - a_3} & , a_3 \leq x < a_4 \\ 0 & , \text{otherwise} \end{cases}$$

The α-cut of $\tilde{\lambda}$ is calculated as

$$\tilde{\lambda}[\alpha] = \{[a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]\}$$

Now the fuzzy function $\tilde{S}(t)[\alpha]$ of the components reliability is obtained as

$$\tilde{S}(t)[\alpha] = \{\int_t^\infty \lambda e^{-\lambda x} dx \mid \lambda \in \tilde{\lambda}[\alpha]\} = \{e^{-\lambda t} \mid \lambda \in \tilde{\lambda}[\alpha]\}$$

Finally, we get

$$\tilde{S}(t)[\alpha] = \{e^{-\{a_4 - \alpha(a_4 - a_3)\}t}, e^{-\{a_1 + \alpha(a_2 - a_1)\}t}\}$$

From the expression of $\tilde{S}(t)[\alpha]$, it can be seen that it is a function of two variables, time (t) and alpha (α).

5. Mathematical formulation of problem

Consider n independent and non-identical components G_j where j=1,2,...,n. Let g_j= 1(0) represent the states: working (failed) of the component j. Also, suppose that there are two types of uncertainties present in the system, namely aleatory and epistemic. In order to remove these uncertainties, one can use the help of mass distribution and the fuzzy reliability theory. In this model, we have combined these two ways and generated a new way to remove the uncertainties existing in the system.

Now, suppose each component j of the system follows fuzzy exponential distribution with fuzzy probability $\tilde{p}_j = [p_{jL}, p_{jU}]$, where j=1, 2,...., n is the reliability interval. To distribute masses among the components, we formulate power set of the state set {0,1}, which is denoted by G ={{0},{1},{0,1}}, and mass distribution corresponding to elements of G function is given by the formula:

$$m_{ji_j} = \begin{cases} 1 - p_{jU}, & G_{ji_j} = \{0\} \\ P_{jL}, & G_{ji_j} = \{1\} \\ 1 - [1 - p_{jU} + P_{jL}], & G_{ji_j} = \{0,1\} \end{cases} \text{, where } G_{ji_j} \in G, i_j = 1,2,3 \text{ and } j = 1,2,....,n. \tag{1}$$

$$\text{Define, } \quad \varphi_{ji_j} = \begin{cases} 1, & 0 \in G_{ji_j} \\ 0, & 0 \notin G_{ji_j} \end{cases}, \quad i_j = 1,2,3 \text{ and } j = 1,2,....,n. \tag{2}$$

$$\psi_{ji_j} = \begin{cases} 1, & 1 \in G_{ji_j} \\ 0, & 1 \notin G_{ji_j} \end{cases}, \quad i_j = 1,2,3 \text{ and } j = 1,2,....,n. \tag{3}$$

$$X = \sum_{j=1}^{n} \varphi_{ji_j} \quad \text{and} \quad Y = \sum_{j=1}^{n} \psi_{ji_j} \tag{4}$$

Here, it is clear that X and Y are the number of failed and working components in the system respectively.
Now, let us present the formulae to calculate BUGF of the system.
If $U_1(z)$ is the belief u–function of component 1, then

$$U_1(z) = \sum_{i_1=1}^{3} m_{1i_1} \, z^{G_{1i_1}, X, Y} \tag{5}$$

If $U_{1,2}(z)$ is the belief u–function of components 1 and 2, then

$$U_{1,2}(z) = \sum_{i_2=0}^{3} \sum_{i_1=0}^{3} \left[\prod_{j=1}^{2} m_{ji_j} \right] z^{(G_{1i_1}, G_{2i_2}), X, Y} \tag{6}$$

Similarly, if $U_{1,2,....,n}(z)$ is the belief u–function of components 1,2,....and n, then

$$U_{1,2,....,n}(z) = \sum_{i_n=0}^{1} \cdots \sum_{i_1=0}^{1} \left[\prod_{j=1}^{n} m_{ji_j} \right] z^{(G_{1i_1}, G_{2i_1}, ..., G_{ni_n}), X, Y} \tag{7}$$

5.1. Reliability evaluation of the considered system by using BUGF

For the survivability of k-out-of-n: G system at least k components must work, and the formula for its reliability function is given by:

$$R(n, k : G) = Ms(Y \geq k) \tag{8}$$

For the survivability of *k*-out-of-*n*: F system, the number of failed components should be less than *k,* and the formula for its reliability function is given by:

$$R(n,k:F)=Ms(X<k) \\ =1-Ms(X\geq k) \tag{9}$$

Here, *Ms* stands for mass.

5.2. Mean time to failure

The mean time to failure is the measure of time between any two consecutive failures. Since the mass of each component depends upon its reliability, it follows an exponential distribution (from equation (1)). From equations (8) and (9), it is clear that the reliabilities are obtained in terms of masses, and ultimately, it can be represented in the form of an exponential distribution. Therefore, the formula for mean time to failure of *k*-out-of-*n*: G (F) systems is demonstrated as

$$\mathrm{MTTF}(n,k:G)=\int_0^\infty R(n,k:G)(t)dt \tag{10}$$

$$\mathrm{MTTF}(n,k:F)=\int_0^\infty R(n,k:F)(t)dt \tag{11}$$

6. Numerical Example

Let us consider 2-out-of-4: G and 2-out-of-4: F systems of non-identical and independent components that have two possible states: working and failed. Let $\tilde{p}_j$ be the fuzzy probability of the success of the component *j* (*j* =1,2,3,4). Diagrammatically the considered system can be shown as depicted in Figure 1.

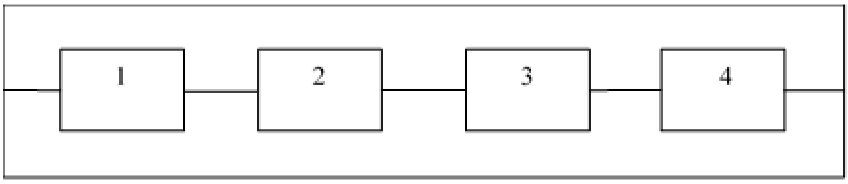

Figure 1. 2-out-of-4: G (F)

Table 1. α-cut intervals of fuzzy failure rates $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$

α	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
0.0	[0.010, 0.090]	[0.350,0.500]	[0.200,0.350]	[0.150,0.300]
0.1	[0.014,0.088]	[0.355.0.495]	[0.205,0.345]	[0.155,0.295]
0.2	[0.018,0.086]	[0.360,0.490]	[0.210,0.340]	[0.160,0.290]
0.3	[0.022,0.084]	[0.365,0.485]	[0.215,0.335]	[0.165,0.285]
0.4	[0.026,0.082]	[0.370,0.480]	[0.220,0.330]	[0.170,0.280]
0.5	[0.030,0.080]	[0.375,0.475]	[0.225,0.325]	[0.175,0.275]
0.6	[0.034,0.078]	[0.380,0.470]	[0.230,0.320]	[0.180,0.270]
0.7	[0.038,0.076]	[0.385,0.465]	[0.235,0.315]	[0.185,0.265]
0.8	[0.042,0.074]	[0.390,0.460]	[0.240,0.310]	[0.190,0.260]

Table 2. α-cut intervals of fuzzy probabilities $\tilde{p}_1, \tilde{p}_2, \tilde{p}_3$ and $\tilde{p}_4$ w.r.t. α

α	$\tilde{p}_1$	$\tilde{p}_2$	$\tilde{p}_3$	$\tilde{p}_4$
0	[0.697676, 0.960789]	[0.135335, 0.246597]	[0.246597, 0.449329]	[0.301194, 0.548812]
0.1	[0.703280, 0.945539]	[0.138069, 0.241714]	[0.251579, 0.440432]	[0.307279, 0.537944]
0.2	[0.708929, 0.930531]	[0.140858, 0.236928]	[0.256661, 0.431711]	[0.313486, 0.527292]
0.3	[0.714623, 0.915761]	[0.143704, 0.232236]	[0.261846, 0.423162]	[0.319819, 0.516851]
0.4	[0.720363, 0.901225]	[0.146607, 0.227638]	[0.267135, 0.414783]	[0.326280, 0.506617]
0.5	[0.7261490, 0.88692]	[0.1495690, 0.22313]	[0.272532, 0.414783]	[0.332871, 0.496585]
0.6	[0.731982, 0.872843]	[0.152590, 0.218712]	[0.278037, 0.398519]	[0.339596, 0.486752]
0.7	[0.737861, 0.858988]	[0.155673,0.214381]	[0.283654, 0.390628]	[0.346456, 0.477114]
0.8	[0.743787, 0.845354]	[0.158817,0.210136]	[0.289384, 0.382893]	[0.353455, 0.467666]

Also suppose that each component follows fuzzy exponential distribution with trapezoidal fuzzy number as the failure rate $\tilde{\lambda}_1 = (0.01, 0.05, 0.07, 0.09)$, $\tilde{\lambda}_2 = (0.35, 0.4, 0.45, 0.5)$, $\tilde{\lambda}_3 = (0.2, 0.25, 0.3, 0.35)$ and $\tilde{\lambda}_4 = (0.15, 0.2, 0.25, 0.3)$ respectively, then α-cut intervals of fuzzy failure rates $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\lambda}_3$ and $\tilde{\lambda}_4$ and the corresponding probabilities with respect to α and time are demonstrated in Tables 1, 2 and 3 respectively.

Table 3. α-cut intervals of fuzzy probabilities $\tilde{p}_1, \tilde{p}_2, \tilde{p}_3$ and $\tilde{p}_4$ w.r.t. time

t	$\tilde{p}_1$	$\tilde{p}_2$	$\tilde{p}_3$	$\tilde{p}_4$
0	[1.000000,1.000000]	[1.000000,1.000000]	[1.000000,1.000000]	[1.000000,1.000000]
1	[0.919431, 0.978240]	[0.615697, 0.694197]	[0.715338, 0.806541]	[0.752014, 0.847894]
2	[0.845354, 0.956954]	[0.379083,0.481909]	[0.511709, 0.650509]	[0.565525, 0.718924]
3	[0.777245, 0.936131]	[0.233400, 0.334540]	[0.366045, 0.524663]	[0.425283, 0.609571]
4	[0.714623, 0.915761]	[0.143704, 0.232236]	[0.261846, 0.423162]	[0.319819, 0.516851]
5	[0.657047, 0.895834]	[0.088478, 0.161218]	[0.187308, 0.341298]	[0.240508, 0.438235]
6	[0.604109, 0.876341]	[0.054476, 0.111917]	[0.133989, 0.275271]	[0.180866, 0.371577]
7	[0.555437, 0.857272]	[0.033541, 0.077692]	[0.095847, 0.222017]	[0.136014, 0.315058]
8	[0.510686, 0.838618]	[0.020651, 0.053934]	[0.068563, 0.179066]	[0.102284, 0.267135]

Since the set of states of each component in the proposed system have two elements, 1 and 0, the power set of state set of each component is the same and is represented by $G = \{\{0\},\{1\},\{0,1\}\}$. Let m_{ij}, where i = 1, 2, 3, 4 and j = 1, 2, 3 are the masses of the components 1, 2, 3 and 4 corresponding to elements 1, 2 and 3 of G respectively. With the help of equation (1), we distributed masses among the components, and their variation with time and alpha as given in Tables 4 and 5.

Table 4. Changes of masses with respect to time

t	m_{11}	m_{12}	m_{13}	m_{21}	m_{22}	m_{23}	m_{31}	m_{32}	m_{33}	m_{41}	m_{42}	m_{43}
0	0.000000	1.000000	0.00000	0.000000	1.000000	0.000000	0.000000	1.000000	0.000000	0.000000	1.000000	0.000000
1	0.021760	0.919431	0.058809	0.305803	0.615697	0.078499	0.193459	0.715338	0.091203	0.152106	0.752014	0.095879
2	0.043046	0.845354	0.111600	0.518091	0.379083	0.102826	0.349491	0.511709	0.138801	0.281076	0.565525	0.153398
3	0.063869	0.777245	0.158886	0.665460	0.233400	0.101139	0.475337	0.366045	0.158618	0.390429	0.425283	0.184288
4	0.084239	0.714623	0.201138	0.767764	0.143704	0.088532	0.576838	0.261846	0.161316	0.483149	0.319819	0.197032
5	0.104166	0.657047	0.238787	0.838782	0.088478	0.07274	0.658702	0.187308	0.15399	0.561765	0.240508	0.197727
6	0.123659	0.604109	0.272232	0.888083	0.054476	0.057441	0.724729	0.133989	0.141282	0.628423	0.180866	0.190711
7	0.142728	0.555437	0.301835	0.922308	0.033541	0.044152	0.777983	0.095847	0.12617	0.684942	0.136014	0.179044
8	0.161382	0.510686	0.327932	0.946066	0.020651	0.033283	0.820934	0.068563	0.110503	0.732865	0.102284	0.164851

Table 5. Variation on masses for different values of alpha

α	m_{11}	m_{12}	m_{13}	m_{21}	m_{22}	m_{23}	m_{31}	m_{32}	m_{33}	m_{41}	m_{42}	m_{43}
0	0.039211	0.697676	0.263113	0.753403	0.135335	0.111262	0.550671	0.246597	0.202732	0.451188	0.301194	0.247617
0.1	0.054461	0.703280	0.242259	0.758286	0.138069	0.103645	0.559568	0.251579	0.188853	0.462056	0.307279	0.230666
0.2	0.069469	0.708929	0.221602	0.763072	0.140858	0.096069	0.568289	0.256661	0.17505	0.472708	0.313486	0.213806
0.3	0.084239	0.714623	0.201138	0.767764	0.143704	0.088532	0.576838	0.261846	0.161316	0.483149	0.319819	0.197032
0.4	0.098775	0.720363	0.180862	0.772362	0.146607	0.081031	0.585217	0.267135	0.147648	0.493383	0.32628	0.180337
0.5	0.113080	0.726149	0.160771	0.77687	0.149569	0.073562	0.59343	0.272532	0.134038	0.503415	0.332871	0.163714
0.6	0.127157	0.731982	0.140861	0.781288	0.15259	0.066122	0.601481	0.278037	0.120482	0.513248	0.339596	0.147157
0.7	0.141012	0.737861	0.121127	0.785619	0.155673	0.058708	0.609372	0.283654	0.106974	0.522886	0.346456	0.130658
0.8	0.154646	0.743787	0.101566	0.789864	0.158817	0.051319	0.617107	0.289384	0.093509	0.532334	0.353455	0.114212

Now, to evaluate the reliability of the system, first we compute the BUGF of its components.

The BUGF of component 1, $U_1(z)$ is computed by equation (5) as

$$U_1(z) = m_{11} z^{\{0\},1,0} + m_{12} z^{\{1\},0,1} + m_{13} z^{\{0,1\},1,1}$$

The BUGF of component 1 and 2, $U_{1,2}(z)$ is obtained by equation (6) as

$$U_{12}(z) = m_{11} m_{21} z^{(\{0\},\{0\}),2,0} + m_{11} m_{22} z^{(\{0\},\{1\}),1,1} + m_{11} m_{23} z^{(\{0\},\{0,1\}),2,1} + m_{12} m_{21} z^{(\{1\},\{0\}),1,1} + m_{12} m_{22} z^{(\{1\},\{1\}),0,2} + m_{12} m_{23} z^{(\{1\},\{0,1\}),1,2} + m_{13} m_{21} z^{(\{0,1\},\{0\}),2,1} + m_{13} m_{22} z^{(\{0,1\},\{1\}),1,2} + m_{13} m_{23} z^{(\{0,1\},\{0,1\}),2,2}$$

The BUGF of component 1, 2 and 3, $U_{1,2,3}(z)$ is computed by equation (7) as

$$
\begin{aligned}
U_{123}(z) = {} & m_{11}m_{21}m_{31}z^{(\{0\},\{0\},\{0\}),3,0} + m_{11}m_{21}m_{32}z^{(\{0\},\{0\},\{1\}),2,1} + m_{11}m_{21}m_{33}z^{(\{0\},\{0\},\{0,1\}),3,1} + m_{12}m_{21}m_{31}z^{(\{1\},\{0\},\{0\}),2,1} + m_{12}m_{21}m_{32} \\
& z^{(\{1\},\{0\},\{1\}),1,2} + m_{12}m_{21}m_{33}z^{(\{1\},\{0\},\{0,1\}),2,2} + m_{13}m_{21}m_{31}z^{(\{0,1\},\{0\},\{0\}),3,1} + m_{13}m_{21}m_{32}z^{(\{0,1\},\{0\},\{1\}),2,2} + m_{13}m_{21}m_{33}z^{(\{0,1\},\{0\},\{0,1\}),3,2} \\
& + m_{11}m_{22}m_{31}z^{(\{0\},\{1\},\{0\}),2,1} + m_{11}m_{22}m_{32}z^{(\{0\},\{1\},\{1\}),1,2} + m_{11}m_{22}m_{33}z^{(\{0\},\{1\},\{0,1\}),2,2} + m_{12}m_{22}m_{31}z^{(\{1\},\{1\},\{0\}),1,2} + m_{12}m_{22}m_{32} \\
& z^{(\{1\},\{1\},\{1\}),0,3} + m_{12}m_{22}m_{33}z^{(\{1\},\{1\},\{0,1\}),1,3} + m_{13}m_{22}m_{31}z^{(\{0,1\},\{1\},\{0\}),2,2} + m_{13}m_{22}m_{32}z^{(\{0,1\},\{1\},\{1\}),1,3} + m_{13}m_{22}m_{33}z^{(\{0,1\},\{1\},\{0,1\}),2,3} \\
& + m_{11}m_{23}m_{31}z^{(\{0\},\{0,1\},\{0\}),3,1} + m_{11}m_{23}m_{32}z^{(\{0\},\{0,1\},\{1\}),2,2} + m_{11}m_{23}m_{33}z^{(\{0\},\{0,1\},\{0,1\}),3,2} + m_{12}m_{23}m_{31}z^{(\{1\},\{0,1\},\{0\}),2,2} + m_{12}m_{23}m_{32} \\
& z^{(\{1\},\{0,1\},\{1\}),1,3} + m_{12}m_{23}m_{33}z^{(\{1\},\{0,1\},\{0,1\}),2,3} + m_{13}m_{23}m_{31}z^{(\{0,1\},\{0,1\},\{0\}),3,2} + m_{13}m_{23}m_{32}z^{(\{0,1\},\{0,1\},\{1\}),2,3} + m_{13}m_{23}m_{33}z^{(\{0,1\},\{0,1\},\{0,1\}),3,3}
\end{aligned}
$$

The BUGF of component 1, 2,3 and 4, $U_{1,2,3,4}(z)$ is obtained by equation (7) as

$$
\begin{aligned}
U_{1234}(z) = {} & m_{11}m_{21}m_{31}m_{41}z^{(\{0\},\{0\},\{0\},\{0\}),4,0} + m_{11}m_{22}m_{31}m_{41}z^{(\{0\},\{0\},\{1\},\{0\}),3,1} + m_{11}m_{23}m_{31}m_{41}z^{(\{0\},\{0\},\{0\},\{0,1\}),4,1} \\
& + m_{12}m_{21}m_{31}m_{41}z^{(\{1\},\{0\},\{0\},\{0\}),3,1} + m_{12}m_{22}m_{31}m_{41}z^{(\{1\},\{1\},\{0\},\{0\}),2,2} + m_{12}m_{23}m_{31}m_{41}z^{(\{1\},\{0,1\},\{0\},\{0\}),3,2} \\
& + m_{13}m_{21}m_{31}m_{41}z^{(\{0,1\},\{0\},\{0\},\{0\}),4,1} + m_{13}m_{22}m_{31}m_{41}z^{(\{0,1\},\{1\},\{0\},\{0\}),3,2} + m_{13}m_{23}m_{31}m_{41}z^{(\{0,1\},\{0,1\},\{0\},\{0\}),4,2} \\
& + m_{11}m_{21}m_{32}m_{41}z^{(\{0\},\{0\},\{1\},\{0\}),3,1} + m_{11}m_{22}m_{32}m_{41}z^{(\{0\},\{1\},\{1\},\{0\}),2,2} + m_{11}m_{23}m_{32}m_{41}z^{(\{0\},\{0,1\},\{1\},\{0\}),3,2} \\
& + m_{12}m_{21}m_{32}m_{41}z^{(\{1\},\{0\},\{1\},\{0\}),2,2} + m_{12}m_{22}m_{32}m_{41}z^{(\{1\},\{1\},\{1\},\{0\}),1,3} + m_{12}m_{23}m_{32}m_{41}z^{(\{1\},\{0,1\},\{1\},\{0\}),2,3} \\
& + m_{13}m_{21}m_{32}m_{41}z^{(\{0,1\},\{0\},\{1\},\{0\}),3,2} + m_{13}m_{22}m_{32}m_{41}z^{(\{0,1\},\{1\},\{1\},\{0\}),2,3} + m_{13}m_{23}m_{32}m_{41}z^{(\{0,1\},\{0,1\},\{1\},\{0\}),3,3} \\
& + m_{11}m_{21}m_{33}m_{41}z^{(\{0\},\{0\},\{0,1\},\{0\}),4,1} + m_{11}m_{22}m_{33}m_{41}z^{(\{0\},\{1\},\{0,1\},\{0\}),3,2} + m_{11}m_{23}m_{33}m_{41}z^{(\{0\},\{0,1\},\{0,1\},\{0\}),4,2} \\
& + m_{12}m_{21}m_{33}m_{41}z^{(\{1\},\{0\},\{0,1\},\{0\}),3,2} + m_{12}m_{22}m_{33}m_{41}z^{(\{1\},\{1\},\{0,1\},\{0\}),2,3} + m_{12}m_{23}m_{33}m_{41}z^{(\{1\},\{0,1\},\{0,1\},\{0\}),3,3} \\
& + m_{13}m_{21}m_{33}m_{41}z^{(\{0,1\},\{0\},\{0,1\},\{0\}),4,2} + m_{13}m_{22}m_{33}m_{41}z^{(\{0,1\},\{1\},\{0,1\},\{0\}),3,3} + m_{13}m_{23}m_{33}m_{41}z^{(\{0,1\},\{0,1\},\{0,1\},\{0\}),4,3} \\
& + m_{11}m_{21}m_{31}m_{42}z^{(\{0\},\{0\},\{0\},\{1\}),3,1} + m_{11}m_{22}m_{31}m_{42}z^{(\{0\},\{1\},\{0\},\{1\}),2,2} + m_{11}m_{23}m_{31}m_{42}z^{(\{0\},\{0,1\},\{0\},\{1\}),3,2} \\
& + m_{12}m_{21}m_{31}m_{42}z^{(\{1\},\{0\},\{0\},\{1\}),2,2} + m_{12}m_{22}m_{31}m_{42}z^{(\{1\},\{1\},\{0\},\{1\}),1,3} + m_{12}m_{23}m_{31}m_{42}z^{(\{1\},\{0,1\},\{0\},\{1\}),2,3} \\
& + m_{13}m_{21}m_{31}m_{42}z^{(\{0,1\},\{0\},\{0\},\{1\}),3,2} + m_{13}m_{22}m_{31}m_{42}z^{(\{0,1\},\{1\},\{0\},\{1\}),2,3} + m_{13}m_{23}m_{31}m_{42}z^{(\{0,1\},\{0,1\},\{0\},\{1\}),3,3} \\
& + m_{11}m_{21}m_{32}m_{42}z^{(\{0\},\{0\},\{1\},\{1\}),2,2} + m_{11}m_{22}m_{32}m_{42}z^{(\{0\},\{1\},\{1\},\{1\}),1,3} + m_{11}m_{23}m_{32}m_{42}z^{(\{0\},\{0,1\},\{1\},\{1\}),2,3} \\
& + m_{12}m_{21}m_{32}m_{42}z^{(\{1\},\{0\},\{1\},\{1\}),1,3} + m_{12}m_{22}m_{32}m_{42}z^{(\{1\},\{1\},\{1\},\{1\}),0,4} + m_{12}m_{23}m_{32}m_{42}z^{(\{1\},\{0,1\},\{1\},\{1\}),1,4} \\
& + m_{13}m_{21}m_{32}m_{42}z^{(\{0,1\},\{0\},\{1\},\{1\}),2,3} + m_{13}m_{22}m_{32}m_{42}z^{(\{0,1\},\{1\},\{1\},\{1\}),1,4} + m_{13}m_{23}m_{32}m_{42}z^{(\{0,1\},\{0,1\},\{1\},\{1\}),2,4} \\
& + m_{11}m_{21}m_{33}m_{42}z^{(\{0\},\{0\},\{0,1\},\{1\}),3,2} + m_{11}m_{22}m_{33}m_{42}z^{(\{0\},\{1\},\{0,1\},\{1\}),2,3} + m_{11}m_{23}m_{33}m_{42}z^{(\{0\},\{0,1\},\{0,1\},\{1\}),3,3} \\
& + m_{12}m_{21}m_{33}m_{42}z^{(\{1\},\{0\},\{0,1\},\{1\}),2,3} + m_{12}m_{22}m_{33}m_{42}z^{(\{1\},\{1\},\{0,1\},\{1\}),1,4} + m_{12}m_{23}m_{33}m_{42}z^{(\{1\},\{0,1\},\{0,1\},\{1\}),2,4} \\
& + m_{13}m_{21}m_{33}m_{42}z^{(\{0,1\},\{0\},\{0,1\},\{1\}),3,3} + m_{13}m_{22}m_{33}m_{42}z^{(\{0,1\},\{1\},\{0,1\},\{1\}),2,4} + m_{13}m_{23}m_{33}m_{42}z^{(\{0,1\},\{0,1\},\{0,1\},\{1\}),3,4} \\
& + m_{11}m_{21}m_{31}m_{43}z^{(\{0\},\{0\},\{0\},\{0,1\}),4,1} + m_{11}m_{22}m_{31}m_{43}z^{(\{0\},\{1\},\{0\},\{0,1\}),2,2} + m_{11}m_{23}m_{31}m_{43}z^{(\{0\},\{0,1\},\{0\},\{0,1\}),4,2} \\
& + m_{12}m_{21}m_{31}m_{43}z^{(\{1\},\{0\},\{0\},\{0,1\}),3,2} + m_{12}m_{22}m_{31}m_{43}z^{(\{1\},\{1\},\{0\},\{0,1\}),2,3} + m_{12}m_{23}m_{31}m_{43}z^{(\{1\},\{0,1\},\{0\},\{0,1\}),3,3} \\
& + m_{13}m_{21}m_{31}m_{43}z^{(0,1\},\{0\},\{0\},\{0,1\}),4,2} + m_{13}m_{22}m_{31}m_{43}z^{(\{0,1\},\{1\},\{0\},\{0,1\}),3,2} + m_{13}m_{23}m_{31}m_{43}z^{(\{0,1\},\{0,1\},\{0\},\{0,1\}),4,3} \\
& + m_{11}m_{21}m_{32}m_{43}z^{(\{0\},\{0\},\{1\},\{0,1\}),3,2} + m_{11}m_{22}m_{32}m_{43}z^{(\{0\},\{1\},\{1\},\{0,1\}),2,3} + m_{11}m_{23}m_{32}m_{43}z^{(\{0\},\{0,1\},\{1\},\{0,1\}),3,3} \\
& + m_{12}m_{21}m_{32}m_{43}z^{(\{1\},\{0\},\{1\},\{0,1\}),2,3} + m_{12}m_{22}m_{32}m_{43}z^{(\{1\},\{1\},\{1\},\{0,1\}),1,4} + m_{12}m_{23}m_{32}m_{43}z^{(\{1\},\{0,1\},\{1\},\{0,1\}),2,4} \\
& + m_{13}m_{21}m_{32}m_{43}z^{(\{0,1\},\{0\},\{1\},\{0,1\}),3,3} + m_{13}m_{22}m_{32}m_{43}z^{(\{0,1\},\{1\},\{1\},\{0,1\}),2,4} + m_{13}m_{23}m_{32}m_{43}z^{(\{0,1\},\{0,1\},\{1\},\{0,1\}),3,4} \\
& + m_{11}m_{21}m_{33}m_{43}z^{(\{0\},\{0\},\{0,1\},\{0,1\}),4,2} + m_{11}m_{22}m_{33}m_{43}z^{(\{0\},\{1\},\{0,1\},\{0,1\}),3,3} + m_{11}m_{23}m_{33}m_{43}z^{(\{0\},\{0,1\},\{0,1\},\{0,1\}),4,3} \\
& + m_{12}m_{21}m_{33}m_{43}z^{(\{1\},\{0\},\{0,1\},\{0,1\}),3,3} + m_{12}m_{22}m_{33}m_{43}z^{(\{1\},\{1\},\{0,1\},\{0,1\}),2,4} + m_{12}m_{23}m_{33}m_{43}z^{(\{1\},\{0,1\},\{0,1\},\{0,1\}),3,4} \\
& + m_{13}m_{21}m_{33}m_{43}z^{(\{0,1\},\{0\},\{0,1\},\{0,1\}),4,3} + m_{13}m_{22}m_{33}m_{43}z^{(\{0,1\},\{1\},\{0,1\},\{0,1\}),3,4} + m_{13}m_{23}m_{33}m_{43}z^{(\{0,1\},\{0,1\},\{0,1\},\{0,1\}),4,4}
\end{aligned}
\tag{12}
$$

The reliabilities R(2, 4: G) of 2-out-of-4: G and R(2, 4: F) of 2-out-of-4: F systems, which can be computed by the equations (8), (9) and (12), are given below:

$$\begin{aligned}R(2,4:\mathrm{G}) = {} & m_{12}m_{22}m_{31}m_{41} + m_{12}m_{23}m_{31}m_{41} + m_{13}m_{22}m_{31}m_{41} + m_{13}m_{23}m_{31}m_{41} + m_{11}m_{22}m_{32}m_{41} \\ & + m_{11}m_{23}m_{32}m_{41} + m_{12}m_{21}m_{32}m_{41} + m_{12}m_{22}m_{32}m_{41} + m_{12}m_{23}m_{32}m_{41} + m_{13}m_{21}m_{32}m_{41} \\ & + m_{13}m_{22}m_{32}m_{41} + m_{13}m_{23}m_{32}m_{41} + m_{11}m_{22}m_{33}m_{41} + m_{11}m_{23}m_{33}m_{41} + m_{13}m_{21}m_{33}m_{41} \\ & + m_{13}m_{22}m_{33}m_{41} + m_{13}m_{23}m_{33}m_{41} + m_{11}m_{22}m_{31}m_{42} + m_{11}m_{23}m_{31}m_{42} + m_{12}m_{21}m_{31}m_{42} \\ & + m_{12}m_{22}m_{31}m_{42} + m_{12}m_{23}m_{31}m_{42} + m_{13}m_{21}m_{31}m_{42} + m_{13}m_{22}m_{31}m_{42} + m_{13}m_{23}m_{31}m_{42} \\ & + m_{11}m_{21}m_{32}m_{42} + m_{11}m_{22}m_{32}m_{42} + m_{11}m_{23}m_{32}m_{42} + m_{12}m_{21}m_{32}m_{42} + m_{12}m_{22}m_{32}m_{42} \\ & + m_{12}m_{23}m_{32}m_{42} + m_{13}m_{21}m_{32}m_{42} + m_{13}m_{22}m_{32}m_{42} + m_{13}m_{23}m_{32}m_{42} + m_{13}m_{21}m_{33}m_{42} \\ & + m_{13}m_{22}m_{33}m_{42} + m_{13}m_{23}m_{33}m_{42} + m_{11}m_{22}m_{31}m_{43} + m_{11}m_{23}m_{31}m_{43} + m_{12}m_{21}m_{31}m_{43} \\ & + m_{12}m_{22}m_{31}m_{43} + m_{12}m_{23}m_{31}m_{43} + m_{13}m_{21}m_{31}m_{43} + m_{13}m_{22}m_{31}m_{43} + m_{13}m_{23}m_{31}m_{43} \\ & + m_{11}m_{21}m_{32}m_{43} + m_{11}m_{22}m_{32}m_{43} + m_{11}m_{23}m_{32}m_{43} + m_{12}m_{21}m_{32}m_{43} + m_{12}m_{22}m_{32}m_{43} \\ & + m_{12}m_{23}m_{32}m_{43} + m_{13}m_{21}m_{32}m_{43} + m_{13}m_{22}m_{32}m_{43} + m_{13}m_{23}m_{32}m_{43} + m_{11}m_{21}m_{33}m_{43} \\ & + m_{11}m_{22}m_{33}m_{43} + m_{11}m_{23}m_{33}m_{43} + m_{12}m_{21}m_{33}m_{43} + m_{12}m_{22}m_{33}m_{43} + m_{12}m_{23}m_{33}m_{43} \\ & + m_{13}m_{21}m_{33}m_{43} + m_{13}m_{22}m_{33}m_{43} + m_{13}m_{23}m_{33}m_{43}\end{aligned} \tag{13}$$

$$\begin{aligned}R(2,4;\mathrm{F}) = {} & 1\text{-}[m_{11}m_{21}m_{31}m_{41} + m_{11}m_{22}m_{31}m_{41} + m_{11}m_{23}m_{31}m_{41} + m_{12}m_{21}m_{31}m_{41} + m_{12}m_{22}m_{31}m_{41} + m_{12}m_{23}m_{31}m_{41} \\ & + m_{13}m_{21}m_{31}m_{41} + m_{13}m_{22}m_{31}m_{41} + m_{13}m_{23}m_{31}m_{41} + m_{11}m_{21}m_{32}m_{41} + m_{11}m_{22}m_{32}m_{41} + m_{11}m_{23}m_{32}m_{41} \\ & + m_{12}m_{21}m_{32}m_{41} + m_{12}m_{23}m_{32}m_{41} + m_{13}m_{21}m_{32}m_{41} + m_{13}m_{22}m_{32}m_{41} + m_{13}m_{23}m_{32}m_{41} + m_{11}m_{21}m_{33}m_{41} \\ & + m_{11}m_{22}m_{33}m_{41} + m_{11}m_{23}m_{33}m_{41} + m_{12}m_{21}m_{33}m_{41} + m_{12}m_{22}m_{33}m_{41} + m_{12}m_{23}m_{33}m_{41} + m_{13}m_{21}m_{33}m_{41} \\ & + m_{13}m_{22}m_{33}m_{41} + m_{13}m_{23}m_{33}m_{41} + m_{11}m_{21}m_{31}m_{42} + m_{11}m_{22}m_{31}m_{42} + m_{11}m_{23}m_{31}m_{42} + m_{12}m_{21}m_{31}m_{42} \\ & + m_{12}m_{23}m_{31}m_{42} + m_{13}m_{21}m_{31}m_{42} + m_{13}m_{22}m_{31}m_{42} + m_{13}m_{23}m_{31}m_{42} + m_{11}m_{21}m_{32}m_{42} + m_{11}m_{23}m_{32}m_{42} \\ & + m_{13}m_{21}m_{32}m_{42} + m_{13}m_{23}m_{32}m_{42} + m_{11}m_{21}m_{33}m_{42} + m_{11}m_{22}m_{33}m_{42} + m_{11}m_{23}m_{33}m_{42} + m_{12}m_{21}m_{33}m_{42} \\ & + m_{13}m_{21}m_{33}m_{42} + m_{13}m_{22}m_{33}m_{42} + m_{13}m_{23}m_{33}m_{42} + m_{11}m_{21}m_{31}m_{43} + m_{11}m_{22}m_{31}m_{43} + m_{11}m_{23}m_{31}m_{43} \\ & + m_{12}m_{21}m_{31}m_{43} + m_{12}m_{22}m_{31}m_{43} + m_{12}m_{23}m_{31}m_{43} + m_{13}m_{21}m_{31}m_{43} + m_{13}m_{22}m_{31}m_{43} + m_{13}m_{23}m_{31}m_{43} \\ & + m_{11}m_{21}m_{32}m_{43} + m_{11}m_{22}m_{32}m_{43} + m_{11}m_{23}m_{32}m_{43} + m_{12}m_{21}m_{32}m_{43} + m_{12}m_{23}m_{32}m_{43} + m_{13}m_{21}m_{32}m_{43} \\ & + m_{13}m_{22}m_{32}m_{43} + m_{13}m_{23}m_{32}m_{43} + m_{11}m_{21}m_{33}m_{43} + m_{11}m_{22}m_{33}m_{43} + m_{11}m_{23}m_{33}m_{43} + m_{12}m_{21}m_{33}m_{43} \\ & + m_{12}m_{22}m_{33}m_{43} + m_{12}m_{23}m_{33}m_{43} + m_{13}m_{21}m_{33}m_{43} + m_{13}m_{22}m_{33}m_{43} + m_{13}m_{23}m_{33}m_{43}]\end{aligned} \tag{14}$$

If we express masses in terms of fuzzy probability, then equations (13) and (14) become

$$\begin{aligned}R(2,4;G) = {} & -2e^{-(\lambda_{1L}+\lambda_{3L}+\lambda_{4L})t} - 2e^{-(\lambda_{1L}+\lambda_{2L}+\lambda_{4L})t} + 3e^{-(\lambda_{1L}+\lambda_{2L}+\lambda_{3L}+\lambda_{4L})t} + e^{-(\lambda_{1L}+\lambda_{2L})t} \\ & + e^{-(\lambda_{2L}+\lambda_{3L})t} + e^{-(\lambda_{1L}+\lambda_{3L})t} + e^{-(\lambda_{3L}+\lambda_{4L})t} + e^{-(\lambda_{2L}+\lambda_{4L})t} + e^{-(\lambda_{1L}+\lambda_{4L})t} \\ & -2e^{-(\lambda_{1L}+\lambda_{2L}+\lambda_{3L})t} - 2e^{-(\lambda_{2L}+\lambda_{3L}+\lambda_{4L})t}\end{aligned} \tag{15}$$

$$\begin{aligned}R(2,4:F) = {} & e^{-(\lambda_{1U}+\lambda_{3U}+\lambda_{4U})t} + e^{-(\lambda_{1U}+\lambda_{2U}+\lambda_{4U})t} + e^{-(\lambda_{2U}+\lambda_{3U}+\lambda_{4U})t} + e^{-(\lambda_{1U}+\lambda_{2U}+\lambda_{3U})t} \\ & -3e^{-(\lambda_{1U}+\lambda_{2U}+\lambda_{3U}+\lambda_{4U})t}\end{aligned} \tag{16}$$

By using these masses, variation of reliability of 2-out-of-4: G (F) from equations (13) and (14) with respect to time (t) and alpha (α) are demonstrated in Tables 6, 7 and displayed in Figures 2, 3, 4 and 5.

Table 6. Reliability v/s time (t)

T	R(2, 4,: G)	R(2, 4: F)
0	1.000000	1.000000
1	0.988649	0.742892
2	0.937387	0.421335
3	0.851513	0.216163
4	0.748068	0.105813
5	0.641433	0.050599
6	0.540660	0.023929
7	0.450334	0.011269
8	0.372060	0.005306

Table 7. Reliability v/s alpha (α)

α	R(2, 4,: G)	R(2, 4: F)
0.0	0.092555	0.795493
0.1	0.096787	0.779705
0.2	0.101204	0.763888
0.3	0.105813	0.748068
0.4	0.110621	0.732268
0.5	0.115637	0.716511
0.6	0.120869	0.700818
0.7	0.126324	0.685208
0.8	0.132011	0.669699

By using equations (10), (11), (15) and (16), the MTTF of considered systems are expressed as

$$\begin{aligned}\mathrm{MTTF}(2,4:G) = &-\frac{2}{\lambda_{1L}+\lambda_{3L}+\lambda_{4L}} - \frac{2}{\lambda_{1L}+\lambda_{2L}+\lambda_{4L}} + \frac{3}{\lambda_{1L}+\lambda_{2L}+\lambda_{3L}+\lambda_{4L}} \\ &- \frac{2}{\lambda_{1L}+\lambda_{2L}+\lambda_{3L}} - \frac{2}{\lambda_{2L}+\lambda_{3L}+\lambda_{4L}} + \frac{1}{\lambda_{1L}+\lambda_{2L}} + \frac{1}{\lambda_{2L}+\lambda_{3L}} \\ &+ \frac{1}{\lambda_{1L}+\lambda_{3L}} + \frac{1}{\lambda_{3L}+\lambda_{4L}} + \frac{1}{\lambda_{2L}+\lambda_{4L}} + \frac{1}{\lambda_{1L}+\lambda_{4L}}\end{aligned} \tag{17}$$

$$\begin{aligned}\mathrm{MTTF}(2,4:F) = &\frac{1}{\lambda_{1U}+\lambda_{3U}+\lambda_{4U}} + \frac{1}{\lambda_{1U}+\lambda_{2U}+\lambda_{4U}} - \frac{3}{\lambda_{1U}+\lambda_{2U}+\lambda_{3U}+\lambda_{4U}} \\ &+ \frac{1}{\lambda_{1U}+\lambda_{2U}+\lambda_{3U}} + \frac{1}{\lambda_{2U}+\lambda_{3U}+\lambda_{4U}}\end{aligned} \tag{18}$$

From equations (17) and (18), changes on MTTF of 2-out-of-4: G (F) with respect to alpha (α) are shown in Table 8 and depicted in Figures 6 and 7 respectively.

Table 8. Variations on MTTF with respect to alpha (α)

α	MTTF(2,4:G)	MTTF(2,4:F)
0.0	8.784664	1.988987
0.1	8.386989	2.018238
0.2	8.027108	2.048380
0.3	7.699699	2.079458
0.4	7.400404	2.111516
0.5	7.125623	2.144601
0.6	6.872351	2.178766
0.7	6.638064	2.214065
0.8	6.420621	2.250557

7. Conclusions

In the proposed work, we considered a non-repairable *k*-out-of-*n*: G (F) system by incorporating aleatory and epistemic uncertainties. To tackle these uncertainties, two methods, namely mass distribution and fuzzy reliability theory, are combined. In this approach, failure rate (λ) is taken as the trapezoidal fuzzy number. Using the trapezoidal fuzzy number ($\tilde{\lambda}$), expression for α-cut of fuzzy failure rate $\tilde{\lambda}$ of every component and corresponding fuzzy reliability function are extracted. With the aid of fuzzy reliability function of components, we have distributed masses to the components, and by using these masses, reliability of the target system is evaluated. This approach is demonstrated through 2-out-of-4: G (F) systems, and the obtained results were explained as:

From Table 6, it is clear that the reliabilities of 2-out-of-4: G (F) decreases with the increment in time. From Table 1, it is observed that the lower and upper bounds of failure rates increase and decrease respectively with the increment of α. Equations (15) and (16) reveal that the reliabilities of 2-out-of 4: G and 2-out-of-4 (F) depends upon the lower and upper bounds of failure rates respectively. Therefore, the reliabilities of 2-out-of 4: G (F) decreases (increases) as α increases, which can be seen in Table 7.

Critical examination of Table 1 along with equations (17) and (18) reveals that the MTTF of 2-out-of 4: G and 2-out-of-4 (F) systems are a function of lower and upper bounds of failure rates respectively. Hence MTTF of 2-out-of 4: G and 2-out-of-4 (F) systems also decreases (increases) with the increment of α as depicted in Table 8.

In the future, this methodology can be applied over any industrial and engineering system that has aleatory and epistemic uncertainty.

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