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Bayesian Reliability Analysis of Exponential Distribution Model under a New Loss Function

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Abstract

Loss function is an important content in Bayes statistical inference. The task of this article is to study the reliability analysis of the exponential model based on a new proposed symmetric loss function. The new proposed loss function is established on the basis of the LINEX asymmetric loss function. Firstly, the Bayes estimation of the parameter is derived under the prior distribution of the parameter based on non-information Quasi prior distribution, and then the admissibility of the estimators are also discussed. Furthermore, this paper puts forward a novel testing procedure to evaluate the lifetime performance of exponential products based on the new derived Bayes estimator. Finally, Monte Carlo statistical simulation and an applicable example are used to illustrate that the new proposed Bayes estimators and testing procedure are effective and feasible.

Keywords: exponential distribution; Bayes estimation; compound LINEX symmetric loss function; lifetime performance index

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1. Introduction

The exponential distribution is a very important lifetime distribution in lifetime and reliability studies. The statistical inference and application related to exponential distribution have received great attention. For example, Basak and Balakrishnan [1] studied the prediction problem of survival time of units from exponential distribution based on step-stress testing censored sample with the help of maximum likelihood (ML) approach. Xia et al. [2] put forward a novel likelihood ratio test on the basis of the Schwarz information criterion for detecting possible bathtub-shaped changes when facing a sequence of exponential distribution. Krishna and Goel [3] discussed the classical and Bayes estimation for exponential distribution under entropy loss function on the basis of randomly censored data. Based on a lifetime performance index, Wu et al. [4] designed two acceptance-sampling plans for the exponential model. Jana et al. [5] studied the Bayes reliability estimation for a stress-strength model related to exponential distribution. For more studies about exponential distribution one can see papers [6-10] and references therein.

Assume that X is a product's lifetime, which follows an exponential distribution and has the following density function:

$$f(x;\theta)=\frac{1}{\theta}e^{-\frac{1}{\theta}x},\quad x>0 \tag{1}$$

Here, $\theta>0$ is the unknown parameter.

Bayesian theory has been widely used in natural science, engineering technology, medicine, environmental science, insurance actuarial science, economy and other fields [11-14]. In Bayesian analysis, the loss function plays a critical role in the final estimation result. Because of the convenience of mathematical treatments, most Bayesian inference results are

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studied under symmetric loss functions, such as the squared error loss function. However, in some situations, such as reliability and lifetime testing field, overestimation and underestimation often have different effects on decision results [15]. Then, the squared error loss function cannot work well. Asymmetric loss functions have been proposed by many scholars. LINEX loss is a well-known asymmetric loss function, and many Bayesian statistical inferences are studied under the LINEX loss function [16-19]. Though several symmetric and asymmetric loss functions have been put forward, it is still necessary for scholars to establish new loss functions [20-21]. Based on the LINEX loss function, Zhang [21] developed a new kind of symmetric loss function, named the compound LINEX symmetric loss function. He pointed out some excellent characteristics of the loss function and also discussed the Bayes estimation problem of parameters of normal distribution and exponential distribution under this loss function. In recent years, references [22-24] studied Bayesian statistical inferences of Poisson distribution, Pareto distribution, and Burr XII distribution under this compound LINEX loss function.

Inspired by reference [21], this paper will propose a new kind of compound LINEX symmetric loss function and then study the Bayesian reliability analysis of exponential distribution under the new proposed loss function.

It is very important for modern enterprises to assess the quality of the products effectively. Process capability indices (PCIs) are easy and effective measurement tools for assembling process performance and potential capability. Montgomery [25] introduced a special PCI C_L , to assess the products' lifetime performance whose life is the-larger-the-better. The index C_L is called the lifetime performance index. This article will also study the Bayes estimation and testing procedure of C_L for exponential products under the new proposed loss function.

The main study of this article is organized as follows:

Section 2 not only recalls some preliminary knowledge about prior distribution and C_L, but also constructs a new compound LINEX loss function. Furthermore, it discusses the properties of the loss function. Some preliminary knowledge about exponential distribution and the lifetime performance index will also be recalled. Section 3 studies the Bayesian estimation of the unknown parameter and C_L. Furthermore, a novel Bayes test of the lifetime performance index will be put forward. In Section 4, Monte Carlo simulations are utilized to demonstrate the performance of the ML estimator and the Bayes estimators, and an applied example shows the effectiveness of the Bayesian test procedure. Finally, Section 5 gives concluding remarks.

2. Preliminary Knowledge

2.1. Prior Distribution

Prior distribution is a key element in Bayesian inference. In most Bayesian statistics, it is often assumed that we can obtain some prior knowledge about the unknown parameter θ through investigating from past experience, simulation experiments, or experts' knowledge. Prior distribution is often used to model this prior knowledge.

Assume that the parameter θ has Quasi-prior distribution, which is a non-informative prior distribution with the following form:

$$\pi(\theta) \propto \frac{1}{\theta^d}, \quad \theta > 0 \tag{2}$$

Here, $d > 0$ is the shape parameter of prior distribution (2). If $d = 0$, then $\pi(\theta) \propto 1$ is a discrete prior distribution. If $d = 1$, then $\pi(\theta) \propto 1/\theta$ is a non-informative prior distribution.

2.2. A New Compound LINEX Symmetric Loss Function

As an important component in Bayesian analysis, choosing an appropriate loss function is an important topic. When one wants to estimate the reliability or failure rate, he/she will find that overestimates will bring greater losses than underestimates. Then, the development of new asymmetric loss functions is very necessary. The LINEX loss function is a very familiar asymmetric loss function, and it has the following form (Basu and Ebrahimi [26]):

$$L_c(\Delta) = e^{c\Delta} - c\Delta - 1, \quad c \neq 0 \tag{3}$$

Here, $\Delta = (\delta - \theta)/\theta$, and δ is an estimator of the parameter θ.

In the following discussion, we will study the Bayes reliability analysis of exponential model under a new established symmetric loss function, which is a compound LINEX loss function. It is constructed as follows:

$$L(\Delta) = L_c(\Delta) + L_{-c}(\Delta) = e^{c\Delta} + e^{-c\Delta} - 2 \quad (4)$$

Where $\Delta = (\delta - \theta)/\theta$ and $c > 0$ is the shape parameter of $L(\Delta)$. Figure 1 illustrates the shape of the new constructed loss function (4) with $c = 1$.

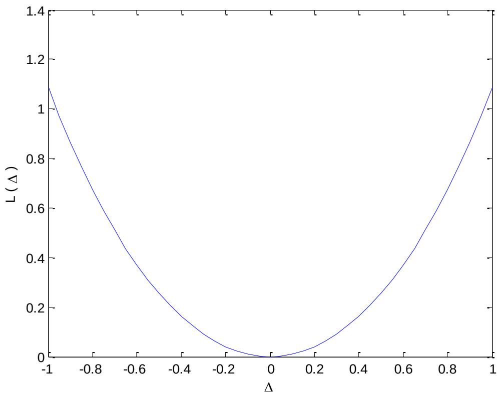

Figure 1. The shape of loss function $L(\Delta)$

From Figure 1 and the mathematical expression of the loss function (4), we know that the new constructed loss function (4) is a symmetric loss function.

Lemma 1 Let δ be an estimator of the parameter θ. $\pi(\theta)$ is a prior distribution of parameter θ, then under the new compound LINEX-based loss (4), the Bayes estimator of θ is the solution of equation (5), which has the following form:

$$e^{-c}E(\frac{1}{\theta}e^{c/\delta} \mid X) = e^{c}E(\frac{1}{\theta}e^{-c/\delta} \mid X) \quad (5)$$

The Bayes estimator is unique, when assuming the Bayes risk $r(\delta) < +\infty$.

Proof Under the new compound LINEX-based loss function (4), the Bayes risk of the estimator δ is:

$$r(\delta) = E_\theta[E(L(\theta,\delta) \mid X)] \quad (6)$$

To make the $r(\delta)$ minimum, we generally only need the minimum of $E[L(\theta,\delta) \mid X]$.

However,

$$E[L(\theta,\delta) \mid X] = E[e^{c(\frac{\delta}{\theta}-1)} + e^{-c(\frac{\delta}{\theta}-1)} - 2 \mid X] = e^{-c}E(\exp\frac{c\delta}{\theta} \mid X) + e^{c}E(\exp\frac{-c\delta}{\theta} \mid X) - 2 \quad (7)$$

Therefore, we only need the right side of the upper formula to be minimal, as shown below:

$$f(\delta) = e^{-c}E(\exp\frac{c\delta}{\theta} \mid X) + e^{c}E(\exp\frac{-c\delta}{\theta} \mid X) - 2 \quad (8)$$

It is obvious that $f''(\delta) > 0$, then the Bayes estimator $\hat{\delta}_B$ of parameter θ satisfies $f'(\delta) = 0$, i.e.,

$$\frac{c}{\theta} \cdot e^{-c} E(\exp\frac{c\delta}{\theta} \mid X) + \frac{c}{\theta} \cdot e^{c} E(\exp\frac{-c\delta}{\theta} \mid X) = 0 \tag{9}$$

Then, we obtain the result (5).

Now, we can prove the uniqueness by demonstrating that $r(\hat{\delta}_B) < +\infty$. According to the assumption $r(\delta) < +\infty$ and the definition of Bayes risk, we have $r(\hat{\delta}_B) < r(\delta)$. Then, $r(\hat{\delta}_B) < +\infty$.

2.3. Lifetime Performance Index of Exponential Product

The application and statistical inference of process capability indices have received great attention [27-30]. For many of the products, consumers want the life of the product to be as long as possible. Montgomery [25] proposed a special process capability index to assess the product, whose lifetime is the larger-the-better. The index is often called the lifetime performance index and has received widespread attention [31-34]. It is defined as

$$C_L = \frac{\mu - L}{\sigma} \tag{10}$$

Here, L is the lower bound of the specifications.

Suppose that X is the lifetime of a product, and it is distributed with exponential distribution (1). Then, we can get $\mu = EX = \theta$ and $\sigma = \sqrt{Var(X)} = \theta$. Now, we can rewrite the index C_L of exponential product as follows:

$$C_L = \frac{\mu - L}{\sigma} = \frac{\theta - L}{\theta} = 1 - \frac{L}{\theta} \tag{11}$$

The failure rate function $r(x)$ can be derived as

$$r(x) = \frac{f(x;\theta)}{1 - F(x;\theta)} = \frac{\theta^{-1}\exp(-\frac{x}{\theta})}{\exp(-\frac{x}{\theta})} = \frac{1}{\theta} \tag{12}$$

From (11) and (12), we know that a larger $1/\theta$ corresponds to a larger C_L and a smaller $r(x)$. Therefore, C_L can be a reasonable and accurate representative tool to assess the product performance.

A conforming rate is an important tool in measuring the product performance, and it is defined as

$$P_r = P(X \geq L) \tag{13}$$

For exponential distribution, we can get

$$P_r = \int_L^{\infty} \theta^{-1} e^{-\frac{x}{\theta}} \mathrm{d}x = e^{C_L - 1}, \ -\infty < C_L < 1 \tag{14}$$

According to equation (14), we can observe that C_L and P_r have a strictly one-to-one increasing relationship. When given the value of C_L, we can easily compute P_r through equation (14), since there exists a one-to-one mapping between P_r and C_L. Therefore, C_L not only is a flexible tool to evaluate products' performance, but also is an effective tool to estimate the conforming rate of a product.

3. Bayes Reliability Analysis of Exponential Distribution

Theorem 1 Assume that $X=(X_1, X_2, \cdots, X_n)$ is a sample drawn from exponential distribution (1), and parameter θ has the Quasi-prior (2). The symbol x is the observation of X, and $x=(x_1, x_2, \cdots, x_n)$. Then, under the new compound LINEX-based loss function (4), the Bayes estimator of the unknown parameter θ is

$$\hat{\theta}_B = \frac{2}{c}\left[\frac{1}{1+exp(-2c/(n+d))} - \frac{1}{2}\right]\cdot T \tag{15}$$

Where $T=\sum_{i=1}^{n} X_i$.

Proof Given sample observation $x=(x_1, x_2, \cdots, x_n)$, the likelihood function of parameter θ is

$$L(x;\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n} \theta^{-1}\exp(-\theta^{-1}x_i) = \theta^{-n}e^{-t/\theta} \tag{16}$$

Here, $t=\sum_{i=1}^{n} x_i$.

Then, the maximum likelihood estimator (MLE) of θ is

$$\hat{\theta}_{ML} = \frac{T}{n} \tag{17}$$

According to equation (16) and the Bayes' Theorem, we can derive the posterior probability density function of parameter θ as follows:

$$h(\theta \mid x) \propto L(x;\theta)\cdot \pi(\theta) \propto \theta^{-n}e^{-t/\theta}\cdot\theta^{-d} \propto \theta^{-(n+d)}e^{-t/\theta} \tag{18}$$

Therefore, $h(\theta \mid x)$ is the density function of the inverse Gamma distribution, noted by $I\Gamma(n+d-1, t)$. That is,

$$h(\theta \mid x) = \frac{t^{n+d-1}}{\Gamma(n+d-1)}\theta^{-(n+d)}e^{-\frac{t}{\theta}} \tag{19}$$

Then, we have

$$E[\frac{1}{\theta}\exp(\frac{c\delta}{\theta}) \mid X] = \int_0^{\infty}\frac{1}{\theta}\exp(\frac{c\delta}{\theta})\cdot h(\theta \mid X)\mathrm{d}\theta = \frac{(n+d-1)T^{n+d-1}}{(T-c\delta)^{n+d}} \tag{20}$$

and

$$E[\frac{1}{\theta}\exp(\frac{-c\delta}{\theta}) \mid X] = \int_0^{\infty}\frac{1}{\theta}\exp(\frac{-c\delta}{\theta})\cdot h(\theta \mid X)\mathrm{d}\theta = \frac{(n+d-1)T^{n+d-1}}{(T+c\delta)^{n+d}} \tag{21}$$

Using equations (20), (21) and (5), we can solve the Bayes estimator of parameter θ as

$$\hat{\theta}_B = \frac{2}{c}\left[\frac{1}{1+exp(-2c/(n+d))} - \frac{1}{2}\right]\cdot T \tag{22}$$

Where $T=\sum_{i=1}^{n}X_i$.

Remark 1 By equation (11), the Bayesian estimator of C_L is

$$\hat{C}_B = 1 - L/\hat{\theta}_B \tag{23}$$

Then, to determine whether the products' lifetime meets a required level, this section will put forward a Bayes testing procedure for C_L.

We need to construct the following hypothesis at first:

$$H_0: C_L \le c_0 \leftrightarrow H_1: C_L > c_0 \tag{24}$$

Let $Y = 2\theta^{-1}T \mid X$, then for given significance level α, according to equation (12), we can easily show that $\theta^{-1} \mid X \sim \Gamma(n+d-1, T)$. Then, $Y \sim \chi^2(2(n+d-1))$. Let $\chi^2_{1-\alpha}(2(n+d-1))$ be the $1-\alpha$ percentile of $\chi^2(2(n+d-1))$. Then,

$$P(2\theta^{-1}T \le \chi^2_{1-\alpha}(2(n+d-1)) \mid X) = 1-\alpha \tag{25}$$

That is,

$$P(\theta^{-1} \le \frac{\chi^2_{1-\alpha}(2(n+d-1))}{2T} \mid X) = 1-\alpha \tag{26}$$

$$P(1-\theta^{-1}L \ge 1 - L\cdot\frac{\chi^2_{1-\alpha}(2(n+d-1))}{2T} \mid X) = 1-\alpha \tag{27}$$

Then, the lower credible limit of C_L with significance level α can be derived as

$$\underline{LB} = 1-(1-\hat{C}_B)\cdot\hat{\theta}_B\frac{\chi^2_{1-\alpha}(2(n+d-1))}{2T} \tag{28}$$

Now, we propose the Bayes testing procedure of C_L as follows:

(i) For given sample size n, under the compound LINEX symmetric entropy loss function (5), calculate the Bayesian estimator $\hat{\theta}_B$ where $T=\sum_{i=1}^{n}X_i$.

(ii) Calculate the $1-\alpha$ one-side credible interval $[\underline{LB}, \infty)$ for lifetime performance index C_L, where $\underline{LB}$ is given in equation (28).

(iii) The rule of the Bayes testing procedure is given as follows:

If $c_0 \notin [\underline{\mathrm{LB}}, \infty)$, then we reject H_0 and conclude that the products' lifetime adheres to the required level. Otherwise, we accept H_0 and conclude that the products' lifetime does not adhere to the required level.

4. Examples

Example 1 (Monte Carlo Simulations) Using the Monte Carlo statistical simulation, we use Matlab software to generate random samples drawn from the exponential distribution (1) with parameter $\theta=1.0$ and sample sizes n=10, 20, 50, 75, 100, respectively. Repeat the simulation experiment N=5000 times and use the mean $\hat{\theta}=\frac{1}{N}\sum_{i=1}^{N}\hat{\theta}_i$ as the estimated value of θ, and use the mean square error $ER(\hat{\theta})=\frac{1}{N}\sum_{i=1}^{N}(\hat{\theta}_i-\theta)^2$ as a measure of good estimation standards. Here, $\hat{\theta}_i$ is the ith estimate of the parameter θ. The estimates and mean square errors of MLE and Bayes estimators are shown in Tables 1 and 2 under different sample sizes for d=0 and 1. The mean square errors are shown in parentheses.

Table 1. Estimates and mean square errors under different sample sizes ($d=0$)

n	10	20	50	75	100
$\hat{\theta}_{ML}$	0.9978 (0.0974)	0.9974 (0.0492)	1.0010 (0.0201)	1.0026 (0.0131)	0.9981 (0.0098)
$\hat{\theta}_B$ (c=0.5)	0.9969 (0.0973)	0.9972 (0.0492)	1.0010 (0.0201)	1.0026 (0.0131)	0.9980 (0.0098)
$\hat{\theta}_B$ (c=1.0)	0.9945 (0.0968)	0.9966 (0.0491)	1.0009 (0.0201)	1.0025 (0.0131)	0.9980 (0.0098)
$\hat{\theta}_B$ (c=1.5)	0.9904 (0.0961)	0.9956 (0.0490)	1.0007 (0.0201)	1.0025 (0.0131)	0.9980 (0.0098)

Table 2. Estimates and mean square errors under different sample sizes ($d=1$)

n	10	20	50	75	100
$\hat{\theta}_{ML}$	1.0029 (0.0997)	0.9975 (0.0492)	0.9973 (0.0198)	0.9986 (0.0128)	1.0010 (0.0102)
$\hat{\theta}_B$ (c=0.5)	0.9111 (0.0902)	0.9498 (0.0471)	0.9777 (0.0196)	0.9855 (0.0127)	0.9910 (0.0101)
$\hat{\theta}_B$ (c=1.0)	0.9092 (0.0902)	0.9493 (0.0471)	0.9776 (0.0196)	0.9854 (0.0127)	0.9910 (0.0101)
$\hat{\theta}_B$ (c=1.5)	0.9061 (0.0902)	0.9484 (0.0471)	0.9774 (0.0196)	0.9853 (0.0127)	0.9910 (0.0101)

From Tables 1 and 2, we can observe that Bayes estimators under compound LINEX-based loss function are affected by the values of c. When sample size n is small, the value of parameter c has a large influence on the estimation results; however, with an increase of n, the values of the mean square error decrease. When n is larger than 50, the influence of it is very small, and sometimes it can be ignored. The Bayes estimate is closer to the true value of θ as n increases. At the same time, we find that when n is large, the change of prior distribution has less influence on the estimation result.

Example 2 (An Application Example) To show the effectiveness and practicable properties of the new Bayes testing procedure, a practical example proposed in Nelson [35] is adopted. Under constant voltage stress, a life testing experiment is done on specimens of a type of electrical insulating fluid. The data set is reported in Table 3.

Table 3. Life testing experiment data

Data Set ($n=19$)									
0.19	0.78	1.31	2.78	0.96	4.15	12.06	6.50	31.75	3.16
4.85	72.89	32.52	4.67	7.35	8.27	8.01	33.91	36.71	

Balakrishnan et al. [36] proved that the life testing data distributed with exponential distribution (1) by using the goodness of fit test combined with the least squares method. We will use this example to illustrate the steps of the new testing procedure of C_L, as follows:

(i) Determine the lower bound L=1.04. That is to say if an electrical insulting fluid's lifetime exceeds L hours, then it is a conforming product. To satisfy the regarding operational performance, P_r is required to exceed some value. Here, we assume the value is 80%. According to equation (9), we know that the value of C_L needs to exceed 0.80. Then, we get the target value c_0 =0.80.

(ii) Establish the following testing hypothesis

$$H_0: C_L \le 0.80 \leftrightarrow H_1: C_L > 0.80 \tag{29}$$

(iii) Specify a significance level $\alpha=0.05$.

(iv) According to equation (28), calculate the 95% credible interval for C_L.

Here, we assume parameter θ has the non-informative prior, i.e., $d=1$. For the loss function (4), the value of the shape parameter is c=1.0. Then, we can obtain $[\underline{\text{LB}},\infty)=[0.8982,\infty)$.

(v) Because $c_0=0.80\notin[\underline{\text{LB}},\infty)$, then we reject H_0 and conclude that the products' lifetime meets the required level.

5. Conclusions

This paper first develops a symmetric loss function based on the most famous asymmetric LINEX loss function. Then, using the Quasi-prior distribution and the new proposed compound LINEX-based loss function, a Bayes estimator of unknown parameter of the exponential distribution is first derived and then a Bayes estimator of lifetime performance index C_L is also obtained. Furthermore, this paper puts forward a new Bayes testing procedure of C_L utilized to evaluate the products' lifetime performance. Application example shows that the new Bayes testing procedure of C_L is easy and effective in evaluating whether the products' lifetime meets the required level.

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